Introduction to Evolution Algebra

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June 28, 2005

Behind the phenomena of genetics and stochastic processes, we find there is an intrinsic algebraic structure. We call it — evolution algebra. Evolution algebras are non-associative, non-power-associative Banach algebras and have many connections with other mathematical fields including graph theory, group theory, Markov chains, dynamic systems, knot theory, 3-manifolds and the study of the Riemann-zeta function. In the present talk, we will introduce the basic concepts of evolution algebras and hierarchical structure theory. One of the unusual features of of an evolution algebra is that it possesses an evolution operator. This evolution operator reveals the dynamic information of an evolution algebra. What makes the theory of evolution algebras different from the classical theory of algebras is that in evolution algebras, we can have two different types of generators: algebraically persistent generators and algebraically transient generators. The basic notions of algebraic persistency and algebraic transiency, and their relative versions, lead to a hierarchical structure on an evolution algebra. Dynamically, this hierarchical structure displays the direction of the flow induced by the evolution operator. Algebraically, this hierarchical structure is given in the form of a sequence of semi-direct-sum decompositions of a general evolution algebra. The dynamic nature of this hierarchical structure is what makes the notion of an evolution algebra applicable to the study of stochastic processes and many other objects in different fields.