When is an n-ary quasigroup an iterated group isotope?

Thomas Zaslavsky

Binghamton University (SUNY), Binghamton, NY, USA

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Rumor has it that Belousov conjectured that an n-ary quasigroup is isotopic to an iterated group if its factorization graph is 3-connected. I have proved this conjecture by a new method employing a kind of branched covering of the factorization graph.

Belousov *et al.* reportedly also proved that an n-ary quasigroup (Q, f), with $n > 2$, is an iterated group isotope if $|Q| \leq 3$; but for $|Q| = 3$ the proof was too long to publish. (I have not been able to find a published proof.) I have a short proof based on the concept of a residual quasigroup of f, that is, a k-ary quasigroup obtained by fixing $n - k$ independent variables in f. If every residual ternary quasigroup is isotopic to an iterated group, then f is isotopic to an iterated group (but that conclusion does not follow if every residual binary quasigroup is an iterated group isotope).