Enveloping algebras of Malcev algebras

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In 2004, Pérez-Izquierdo and Shestakov extended the Poincaré-Birkhoff-Witt theorem from Lie algebras to Malcev algebras: for any Malcev algebra M (over a field of characteristic $\neq 2, 3$), they constructed a universal nonassociative enveloping algebra $U(M)$, which shares many of the attractive properties of the associative enveloping algebras of Lie algebras. In particular, $U(M)$ has a monomial basis of PBW type and a natural Hopf algebra structure. In part one of this talk, I will review Pérez-Izquierdo and Shestakov's construction of $U(M)$. In part two, I will describe how this construction can be used to compute explicit structure constants for the enveloping algebras of low-dimensional Malcev algebras. These computations can be simplified using differential operators on the associated graded algebra of $U(M)$ (a polynomial algebra in $\dim M$ variables) and derivations defined by two elements of M (which is contained in the generalized alternative nucleus of $U(M)$). Since $U(M)$ is not alternative in general, it is also of interest to calculate its maximal alternative quotient, which is the universal alternative enveloping algebra $A(M)$. In part three, I will describe the resulting new examples of infinite dimensional alternative algebras. This program has been carried out for the 4-dimensional (solvable) Malcev algebra and for the 5-dimensional nilpotent Malcev algebra, and is currently underway for the 5-dimensional non-solvable Malcev algebra (which is the semidirect product of the simple 3-dimensional Lie algebra and its unique non-Lie module of dimension 2). The next step is to study the one-parameter family of 5-dimensional solvable Malcev algebras. The ultimate goal of this research program is to calculate the structure constants for $A(M)$ where M is the 7-dimensional simple Malcev algebra; this will be the "universal alternative envelope" of the octonions. This is joint work with Irvin Hentzel, Luiz Peresi, Marina Tvalavadze and Hamid Usefi.