Abstracts of Talks

Third Mile High Conference on Nonassociative Mathematics
August 11–17,2013
University of Denver, Denver, Colorado

(updated August 9, 2013)

Representation groups of finite Osborn loops

*J.O. Adeniran, A.A.A. Agboola and A.O. Isere Abednego**
Ambrose Alli University, Ekpoma, Nigeria

It is shown that an Osborn loop of order n has $n/2$ generators. Given the generators, the representation Π is generated by $R(2) \circ R(2+i) = R(3+i) \forall i = 1, 3, 5, ..., n-3$. The representation of Osborn loops of order 16 is presented and it is used as an example to verify the results. It is also shown that the order of every element of the representation Π divides the order of the loop, hence, Osborn loops of order 16 are langrangelike.

Dihedral-like constructions of automorphic loops

Mouna Aboras
University of Denver

Automorphic loops are loops in which all inner mappings are automorphisms. We study a generalization of the dihedral construction for groups. Namely, if $(G, +)$ is an abelian group, $m \geq 1$ and $\alpha \in \mathrm{Aut}(G)$, let $\mathrm{Dih}(m, G, \alpha)$ on $\mathbb{Z}_m \times G$ be defined by

$$(i, u)(j, v) = (i + j,\ ((-1)^j u + v)\alpha^{ij}).$$

The resulting loop is automorphic if and only if $m = 2$ or ($\alpha^2 = 1$ and m is even). The case $m = 2$ was introduced by Kinyon, Kunen, Phillips, and Vojtěchovský. We present several structural results about the automorphic dihedral loops in both cases.

Eight-dimensional absolute valued algebras

Seidon Alsaody
Uppsala University, Sweden

An absolute valued algebra is a non-zero real algebra endowed with a multiplicative norm. Historical examples are the algebras of real numbers, complex numbers, quaternions and octonions in dimensions 1, 2, 4 and 8, respectively. In 1947, A. A. Albert showed that finite-dimensional absolute valued algebras exist only in these dimensions. They have been classified up to isomorphism in dimension at most four, while in dimension eight, the classification problem is yet unsolved.

In this talk we will give a description of eight-dimensional absolute valued algebras, and some recent results on their structure. These results provide an approach to the classification problem.

Towards a geometric theory for left loops

Karla Baez
Universidad Autonoma del Estado de Morelos, Mexico

In Group Theory there has been a lot of research on the properties of groups given the geometric and combinatorial properties of their Cayley graphs. More recently, thanks to the works of mathematicians like G. Sabidoussi, G. Gauyacq and E. Mwambené, it has been possible to define and study the Cayley graphs of more general structures. These authors gave characterizations of the Cayley graphs of groups, quasigroups and loops respectively. Mwambené has also given a characterization of vertex-transitive graphs as Cayley graphs of left loops with respect to a set with a special characteristic called "quasi-associativity". The problem is that even with these characterization, certain obvious results for groups are false in general. For example, there are loops such that their Cayley graph with respect to a generating set is not connected. Anyway, in this talk will be shown that, if the set it required to be quasi-associative then the basic theorems of group theory apply also for left loops, allowing to define such concepts as a *hyperbolic left loop* or the *rate of growth of a left loop.*

Extensions of nilpotent Lie algebras of type 2

Pilar Benito and Daniel de-la-Concepción*
Universidad de la Rioja, Spain

The study and description of the structure of complex Lie algebras with nilradical a nilpotent Lie algebra of type 2 can be attacked by using $\mathfrak{sl}_2(\mathbb{C})$-representation theory. Our results will be applied to review the classifications given in [5] and [1] of the Lie algebras with nilradical the 3-dimensional Heisenberg Lie algebra and the quasiclassical free nilpotent algebra $\mathfrak{n}_{2,3}$. The 5-dimensional Lie algebra $\mathfrak{n}_{2,3}$ is one of two free nilpotent algebras admitting an invariant metric as it is proven in [2]. According to [4], quasiclassical algebras let construct consistent Yang-Mills gauge theories.

[1] J.M. Ancochea-Bermúdez, R. Campoamor-Stursberg, L. García Vergnolle: Classification of Lie algebras with naturally graded quasi-filiform nilradicals, J. Geometry Phys. 61 (2011), 2168-2186.

[2] V.J. Del Barco, G.P. Ovando: Free nilpotent Lie algebras admitting ad-invariant metrics, arXiv:1104.4773v2.

[3] P. Benito, D. de-la-Concepción: On Levi extensions of nilpotent Lie algebras, Linear Alg. Appl. 439 (5) (2013), 1441-1457.

[4] S. Okubo: Gauge theory based upon solvable Lie algebras, J. Phys. A: Math. Gen. 31 (1998), 7603-7609.

[5] J.L. Rubin, P. Winternitz: Solvable Lie algebras with Heisenberg ideals, J. Phys. A: Math. Gen. 26 (5) (1993) 1123-1138.

[6] T. Sato: The derivations of the Lie algebras, Tohoku. Math. Journ., 23 (1971) 21-36.

Triality

Georgia Benkart
University of Wisconsin-Madison, USA

This talk will focus on Lie algebras with triality and groups with triality and a general framework for unifying their study.

On Loday's parametrized one-relation algebras

Murray Bremner and Vladimir Dotsenko*
University of Saskatchewan, Canada

Loday's parametrized one-relation algebras are defined by a bilinear operation satisfying this polynomial identity for some scalars x_σ ($\sigma \in S_3$):

$$(ab)c \equiv \sum_{\sigma \in S_3} x_\sigma a^\sigma (b^\sigma c^\sigma).$$

It is an open problem to classify the cases satisfying condition S: the corresponding operads are isomorphic as S-modules to the associative operad, and hence the free algebras are isomorphic as graded vector spaces to the (non-unital) tensor algebra. Livernet and Loday studied the one-parameter family of solutions containing associative and Poisson algebras. We study some new solutions: the family $(ab)c \equiv \lambda c(ab)$ ($\lambda \neq \pm 1$), a one-parameter family of deformations of the Leibniz operad, and the dual family of deformations of the Zinbiel operad.

Counting nilpotent loops up to isotopy

Lucien Clavier
Cornell University, USA

We will present tools introduced by Daniel Daly and Petr Vojtěchovský for counting nilpotent loops, and explain how to modify these tools to count nilpotent loops up to isotopy rather than isomorphy.

Mutually orthogonal latin squares: Packing and covering analogues

Charles Colbourn
Arizona State University, USA

Starting with Euler, the existence of sets of mutually orthogonal latin squares has been a fascinating study with connections to algebra, finite geometry, combinatorial designs, number theory, and applications in communications, statistical design, networking, coding theory, and computation. Still the existence questions remain far from being settled! Recently many applications have surfaced in which one wants a good "approximation" to a set of MOLS. A number of these relax the requirements on the squares, and require that when two squares are superimposed either every pair appears at least once (covering) or at most once (packing).

After a quick tour of what is known about MOLS, we explore two problems involving a covering analogue and a packing analogue. The covering variant arises in a combinatorial object called a covering array, while the packing variant arises in a construction of perfect hash families. We describe the connections with MOLS, and discuss some challenging open questions.

Moufang loops
Piroska Csörgő
Eszterhazy Karoly College, Eger, Hungary

Some results in connection with the Phillips' problem: Is there a Moufang loop of odd order with trivial nucleus?

Pattern avoidance in latin squares
Dan Daly, Mike Earnest, Anant Godbole and Sam Gutekunst*
Southeast Missouri State University, USA

Pattern avoidance in permutations has been an important area of study in combinatorics for the last thirty years. We will discuss one generalization of the concept of pattern avoidance to latin squares and discuss enumeration results that follow.

Matter universe
Geoffrey Dixon

Expanding the theory of Dirac spinors (and algebra) to a model based on $T = \mathbb{C} \otimes \mathbb{H} \otimes \mathbb{O}$ leads naturally to the conclusion that our observable (1,3)-spacetime must be dominantly matter, or anti-matter, and that both must exist.

Cyclic extensions of Moufang loops
Aleš Drápal
Charles University in Prague, Czech Republic

Gagola's construction of cyclic extensions gives a Mofang loop under certain conditions. I will specify these conditions, show what this produces for Moufang loops that extend groups and demonstrate that upon examples.

Magic squares of Lie groups
Tevian Dray, John Huerta, Joshua Kincaid, Corinne A. Manogue, Aaron Wangberg and Robert A. Wilson*
Oregon State University, USA

The Tits-Freudenthal magic square yields a description of certain real forms of the exceptional Lie algebras in terms of a pair of (possibly split) division algebras. At the group level, the first two rows are well understood, including a geometric understanding of the minimal representations of F_4 and E_6 in terms of the Albert algebra. In the third row, the minimal representation of E_7 consists of Freudenthal triples.

We present here several results at the group level: A complete description of the corresponding "2×2" magic square as $SU(2, \mathbb{K}' \otimes \mathbb{K})$, the use of Cartan decompositions involving all 5 real forms of E_6 to identify chains of real subgroups of the particular real form $SL(3, \mathbb{O})$, and a new description of Freudenthal triples in terms of "cubies", the components of an antisymmetric rank-3 representation of (generalized) symplectic groups, thus providing a unified, geometric interpretation of Freudenthal triples as a single object, and a new description of the minimal representation of E_7.

In future work, we hope to extend this construction to the fourth row, ultimately providing a unified description of the full magic square.

Fine gradings and gradings by root systems on simple Lie algebras

Alberto Elduque
University of Zaragoza, Spain

The fine gradings by abelian groups on the simple finite dimensional Lie algebras over an algebraically closed field of characteristic zero will be shown to be closely related to the gradings by (not necessarily reduced) root systems, and certain gradings on the coordinate algebras.

A class of latin squares derived from finite abelian groups

Anthony Evans
Wright State University, U.S.A.

We consider latin squares obtained by extending the Cayley tables of finite abelian groups, and give preliminary results on the existence/nonexistence of latin squares orthogonal to these.

Non-associative geometry and the spectral action principle

*Latham Boyle and Shane Farnsworth**
Perimeter Institute, Canada

Non-commutative Spectral Geometry has proven to be a remarkably well-suited framework for describing the standard model of particle physics (coupled to Einstein gravity). I will explain our recent efforts to extend this framework from non-commutative to non-associative geometries. From the physics standpoint, one of the main motivations is in exploring beyond the standard model physics – particularly the so-called grand unified theories. (arXiv:1303.1782)

Generators of certain inner mapping groups

Stephen Gagola III
Charles University in Prague, Czech Republic

A subloop of a loop Q is said to be normal if it is stabilized by all maps in the inner mapping group of Q. But in many cases, the inner mapping group of a Moufang loop is actually generated by conjugation maps. In such cases, subloops are normal if and only if they are stabilized by the conjugation maps. This includes any Moufang loop whose cubes generate either the entire loop or a subloop of index three. f

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Non-Moufang loops satisfying Moufang's theorem

*Giliard Souza dos Anjos and Maria Giuliani**
Federal University of ABC, Brazil

Here I will talk about certain types of loops that are not Moufang but do satisfy Moufang's Theorem.

Wedderburn's principal theorem for Jordan superalgebras with unity

Faber Gomez Gonzalez
Universidad de Antioquia, Colombia

We consider unital finite dimensional Jordan superalgebras J, with solvable radical N and such that $N^2 = 0$ and J/N is simple Jordan superalgebra of some of the following type: superform, D_t or Kac or is of type $K_3 + F1$. We proved that an analogue to the Wedderburn's Principal Theorem (WPT) is valid when some restrictions are imposed on the types of irreducible summands in the Jordan bimodule N. That the restrictions imposed are essential is shown with counter-examples.

Simple right conjugacy closed loops

Mark Greer
University of North Alabama, USA

A loop Q is a *right conjugacy closed loop* (or RCC loop) if R_Q is closed under conjugation. Though, most of the literature on the one-sided conjugacy closed loops deals with left conjugacy closed loops, RCC loops are the more natural choice here since our permutations act on the right. In this talk we give the first general construction of a large class of nonassociative, finite simple RCC loops. Our construction by no means accounts for all such loops; for example, Nagy's Bol loop of exponent 2 does not fit this construction. Thus a full classification of finite simple RCC loops is still elusive. Nevertheless, we have found by exhaustive computer search that our construction accounts for all finite simple RCC loops up to order 15.

Peirce graded algebras of Jordan type

Jonathan Hall, Felix Rehren and Sergey Shpectorov*
Michigan State University, USA

Griess-Majorana-Miyamoto algebras (associated with vertex operator algebras) and Jordan algebras can be generated by idempotents whose associated Peirce decompositions are graded by the group of order 2. With appropriate additional conditions, we classify all such algebras generated by two idempotents. As a consequence, in the non-Jordan case the algebra is always a quotient of the natural permutation module for a 3-transposition group, no matter how many generators.

G_2 and the rolling ball

*John Baez and John Huerta**
Instituto Superior Tecnico, Lisbon, Portugal

The search for simple models of the exceptional Lie groups is a long standing problem in mathematics. In this talk, we use the split octonions to explain how the smallest exceptional Lie group, G_2, can be thought of as the symmetry group of a 'spinorial ball' rolling on a projective plane precisely 3 times as big.

Nuclear semidirect product of commutative automorphic loops

*Jan Hora and Přemysl Jedlička**
Czech University of Life Sciences, Czech Republic

An automorphic loop is a loop with all inner mappings being automorphisms. A semidirect product of groups is, on one hand, a group G with two subgroups $H < G$ and $K \triangleleft G$ such that $KH = G$ and $K \cap H = 1$. On the other hand, it is a construction of a group G from two groups K and H and a homomorphism $\varphi : H \to \mathrm{Aut}(K)$.

Now consider Q, a commutative automorphic loop. If $H < Q$ and $K \triangleleft Q$ such that $KH = Q$ and $K \cap H = 1$, it is reasonable to speak about a semidirect product of commutative automorphic loops. However, to obtain a construction simple to describe, it is useful to assume two additional conditions: K and H are abelian groups, and $K \leqslant N_\mu(Q)$. In this case we speak about a nuclear semidirect product.

This situation can be described as a construction too, using some mapping $\varphi : H^2 \to \mathrm{Aut}(K)$, satisfying some conditions. We show several examples and we notice that one of the ways how to fulfill the conditions is using φ bilinear. This case is studied deeper and we describe how to obtain such mappings.

Loop determinants and loop S-rings

Kenneth Johnson
Penn State Abington, USA

I will talk about some ideas which have arisen from new (and old) work related to group determinants. Among other things I will talk about loop determinants over finite fields, rings coming from orbits of inner mapping groups and the information in the loop determinant.

Totally automorphic loops

Michael Kinyon
University of Denver, USA

The multiplication group $\mathrm{Mlt}(Q)$ of a loop Q is the group generated by the left and right multiplications L_x, R_x, $x \in Q$, where $L_x y = R_y x = xy$. The inner mapping group $\mathrm{Inn}(Q)$ is the stabilizer in $\mathrm{Mlt}(Q)$ of the identity element 1. A loop is automorphic if every inner mapping is an automorphism.

The recent work of Stanovský and Vojtěchovský on commutator theory for loops (which they will discuss in Friday's morning session) has drawn attention to a larger permutation group, the *total multiplication group* $\mathrm{TMlt}(Q)$ which is generated by the L_x's, the R_x's and the division maps M_x where $M_x y = y\backslash x$. The *total inner mapping group* $\mathrm{TInn}(Q)$ is the stabilizer in $\mathrm{TMlt}(Q)$ of 1.

Call a loop Q *totally automorphic* if every mapping in $\mathrm{TInn}(Q)$ is an automorphism of Q. It is easy to see that every totally automorphic loop is commutative, but in fact, one can say quite a bit more. After a general introduction to automorphic loops (both for my benefit and the benefit of the two speakers to come after me), I will give a complete characterization of totally automorphic loops. It turns out that the variety of totally automorphic loops coincides with a very familiar variety, and the astute in the audience will probably guess what that variety is by the time I reach the end of the talk.

Cayley-Dickson loops and their permutation groups

Jenya Kirshtein

The Cayley-Dickson loop Q_n is the multiplicative closure of basic elements of the algebra constructed by n applications of the Cayley-Dickson doubling process (the first few examples of such algebras are real numbers, complex numbers, quaternions, octonions, sedenions). We discuss the subloop structure of the Cayley-Dickson loops and describe their automorphism groups, multiplication groups, and inner mapping groups.

Sign matrices for frames of 2^n-ons under Smith-Conway and Caley-Dickson multiplication

Benard Kivunge
Kenyatta University, Kenya

There has been a great desire to develop doubling formulas that give better algebraic structures as the dimensions of the algebras so formed increase. Whenever these doubling formulas are applied, several interesting loop and algebraic properties are observed on the structures so formed. The Cayley-Dickson formula is given by $(a,b)(c,d) = (ac - \overline{d}b, da + b\overline{c})$ while the Smith-Conway doubling formula is

$$(a,b)(c,d) = \begin{cases} (ac, \overline{a}d), \text{if b=0;} \\ \left(ac - \overline{\overline{b}d}, b\overline{c} + b\overline{\left(\overline{a}.\overline{b^{-1}d}\right)}\right), \text{if b} \neq 0. \end{cases}$$

A Hadamard matrix of order n is a nXn matrix with entries ± 1 such that $HH^T = nI_n$ where I_n is the identity nXn matrix. It is shown that the sign matrices for the frame multiplication under the Smith-Conway and Cayley-Dickson multiplications are Hadamard matrices. Kronecker products are also introduced, and it is shown that the sign matrices for the quaternion and octonion frames are equivalent to Kronecker products.

Beyond the complexes: Toward a lattice-based number system

Jens Köplinger and John A. Shuster*

We give an interim report on progress towards number systems defined using lattices, following the vision sketched in [1]. Different morphisms are shown, some of which have subspaces that correspond to subspaces of conventional and p-adic number addition, multiplication, and exponentiation. Lattices include the square lattice in the plane, and the integral octonions on the E_8 lattice.

[1] J. Köplinger, J. A. Shuster, $1+1=2$*; A step in the wrong direction?*, FQXi essay contest (2012), http://www.fqxi.org/community/forum/topic/1449

Algebras with operators, Koszul duality, and conformal algebras

Pavel Kolesnikov
Sobolev Institute of Mathematics, Novosibirsk State University, Novosibirsk, Russia

In this talk we will observe and discuss relations between several classes of algebraic systems that appeared in areas including algebraic topology, homological algebra, K-theory, combinatorics, and mathematical physics. A brief sketch of the picture is shown in the table below.

Conformal algebras				(?)		
Algebras & averaging operators				Algebras & Rota–Baxter operators	—	Classical Yang–Baxter equations
$\mathcal{O} \circ$ Perm	—	Operad $\mathcal{O}$	—	$\mathcal{O} \bullet$ pre-Lie		
Loday algebras (dialgebras)	—	**Algebras**	—	**Dendriform algebras (pre-algebras)**		
Perm (left-comm.)	—	Com	—	Zinbiel (dual to Leibniz)		
Leibniz	—	Lie	—	pre-Lie (left-symmetric)		

There are three main columns that can be informally labeled as related to the classes of "ordinary" algebras (central column), Loday algebras also called dialgebras (left column), and dendriform algebras (right column). The left and the right columns present two natural ways of "replicating" the operads governing the classes of ordinary algebras, let us call them both as replicated classes.

As a model example one may consider the class Lie of all Lie algebras: The corresponding replicated classes of algebras are known as Leibniz and pre-Lie (also called left-symmetric) algebras.

In the binary quadratic case, one may observe Koszul duality between the left and right columns, which is very natural since the corresponding replicated operads may be obtained as white and black Manin products with Koszul-dual operads Perm (governing the class of left-commutative algebras) and pre-Lie, respectively. We observe a general method how to deduce the identities defining the replicated classes for an arbitrary operad, not necessarily binary or quadratic.

Further, we consider a relation between algebras in these replicated classes and ordinary algebras with additional structures. The idea of Koszul duality works here and provides a way to embed a Loday algebra into an ordinary algebra with an averaging operator, i.e., a linear map T such that

$$T(x)T(y) = T(xT(y)) = T(T(x)y).$$

Similarly, a dendriform algebra may be embedded into an ordinary algebra with a Rota–Baxter operator, i.e., a linear map R such that

$$R(x)R(y) = R(xR(y)) + R(R(x)y).$$

The right column of the table seems to be more interesting from the algebraic point of view. For example, pre-Lie algebras (dendriform Lie algebras) have an extremely wide area of applications, Rota–Baxter operators appeared initially in fluctuation theory (by G. Baxter) and later in probability theory (G.-C. Rota), they turn to be closely related with the classical Yang–Baxter equation (after C. Yang and R. Baxter). This is why the algebraic study of dendriform algebras and related structures is an actual and even urgent topic of research.

In contrast, the left column (Leibniz algebras and around) have less applications. Notably, there is a general way how to solve algebraic problems for these structures by means of conformal algebras. The latter were introduced by V. Kac as a tool in the study of vertex operator algebras: The singular part of an operator product expansion describes the commutator of two chiral fields in 2-dimensional conformal field theory. Every Loday algebra may be naturally embedded into an appropriate conformal algebra. This observation allows to reduce many problems concerning this replication to the case of ordinary algebras. Unfortunately, a “dual analogue” of conformal algebras, which may work in the same way for dendriform algebras, is unknown.

As an example, we consider analogues of the Poincare–Birkhoff–Witt and Ado theorems for Lie algebras. These statements are known to be true for Leibniz algebras but remain unclear in the dendriform settings (for pre-Lie algebras).

Some gradings of nonassociative algebras related to fine gradings of exceptional simple Lie algebras

*Alberto Elduque and Mikhail Kotchetov**
Memorial University of Newfoundland, Canada

Gradings on Lie algebras by various abelian groups arise in the theory of symmetric spaces, Kac-Moody algebras, and color Lie superalgebras. In the 1960s, V. Kac classified all gradings by cyclic groups on finite-dimensional simple Lie algebras over complex numbers. In the past two decades, there has been much progress in the study of gradings on simple Lie algebras by arbitrary groups. In particular, over an algebraically closed field of characteristic zero, fine gradings have been classified on all finite-dimensional simple Lie algebras except E_7 and E_8.

In a recent paper, A. Elduque has shown that, in a sense, a fine grading splits into two independent gradings: a grading by a free abelian group, which is also a grading by a root system, and a fine grading by a finite group on the corresponding coordinate algebra. For example, the $\mathbb{Z}_2^3$-grading on the algebra of octonions that arises from the three iterations of Cayley-Dickson doubling process is "responsible" not only for a fine $\mathbb{Z}_2^3$-grading on G_2 but also for fine gradings on F_4 by $\mathbb{Z} \times \mathbb{Z}_2^3$ and on E_r by $\mathbb{Z}^{r-4} \times \mathbb{Z}_2^3$ ($r = 6, 7, 8$). We have a similar picture for the $\mathbb{Z}_3^3$-grading on the simple exceptional Jordan algebra that can be obtained from the first Tits construction.

In this talk, we will discuss the $\mathbb{Z}_4^3$-grading on the simple structurable algebra of dimension 56, the double of the Jordan algebra of Hermitian matrices of order 4 over quaternions. This grading with the $\mathbb{Z}_2^3$ and $\mathbb{Z}_3^3$-gradings mentioned above can be used to construct almost all known fine gradings on exceptional simple Lie algebras.

Octonionic ovoids

Eric Moorhouse
University of Wyoming, USA

Conway, Kleidman and Wilson (1988) and the speaker (1993) have used the octonions to construct new ovoids in $O_8^+(p)$, the triality quadric in projective 7-space over a field of prime order p. (An *ovoid* in such a quadric is a set of p^3+1 points of the quadric, no two on a line of the quadric.) Throughout its 25-year history, this connection has continued to yield many interesting insights; but many open problems remain. We will survey some highlights of these.

The upper triangular algebra loop of degree 4

K.W. Johnson, Michael Munywoki and J.D.H. Smith*
Iowa State University

A natural loop structure is defined on the set U_4 of unimodular upper-triangular matrices over a given field. Inner mappings of the loop are computed. It is shown that the loop is non-associative and nilpotent, of class 3. A detailed listing of the loop conjugacy classes is presented.

Finite simple automorphic loops

*Alexander Grishkov, Michael Kinyon and Gábor Nagy**
University of Szeged, Hungary

A loop Q is said to be an *automorphic loop,* or briefly an A-loop, if all inner mappings are automorphisms. The question of the existence of nonassociative (finite) simple A-loops is one of the most exciting open problems in recent loop theory. Many partial results are due to Jedlička, Vojtěchovský, Kinyon, Phillips, Kunen, Johnson and others. I will present the proof of the nonexistence of finite simple commutative A-loops. The proof is rather nice mathematics; it includes computer-aided formal computations, deep results from the theory of finite groups, and the theory of finite Lie rings.

Nim addition and split extensions basis for 2^n-ons

*Benard Kivunge and Lydia Nyambura Njuguna**
Kenyatta University, Nairobi, Kenya

Hypercomplex numbers are numbers which are obtained by extending complex numbers using various doubling formulae. They include quaternions, octonions, sedenions and the general 2^n-ons. Consider the non-negative numbers $Z^+ = \{0, 1, 2, 3, \cdots\}$. Nim addition gives a way of defining addition in Z^+ to make it a field of characteristic 2. In this paper we perform the multiplication of basis elements of complex, quaternion, octonion and sedenion split extensions using the Jonathan Smith formula. In each case we show that the multiplication is related to Nim addition. We also show that the multiplication of split extensions for general 2^n-ons can be viewed in terms of Nim addition.

Keywords: Hypercomplex numbers, quaternions, octonions, sedenions, spit extensions, Nim addition.

On Crypto-automorphism of quasigroups and loops

*J.O. Adeniran and Y.T. Oyebo**
Lagos State University, Ojo, Lagos, Nigeria

In this paper, the complete structural investigation of the Crypto-Automorphism of quasigroups and loops as a bijection are discussed. Relationship between the bijection, pseudo-automorphism and automorphism are established, these we obtained using the concepts of autotopisms and derivatives. It was shown that, if (A, B, C) is an autotopism of a loop Q, then $C^{-1} \in CUM(Q, \cdot)$ with companions $a = 1B$ and $b = 1A$. It was also shown that, if every pair of elements $a, b \in Q$ is companions of crypto-automorphism, then Q is a $G-$loop. Also, the inner mappings of some Bol-Moufang type of loops are characterized using this bijection.

Keywords: Buchsteiner loop, IP-loops, automorphism group, crypto-automorphism, derivatives.
MSC: Primary 20NO5; Secondary 08A05.

The structure of conjugacy closed loops

*Michael Kinyon and J.D. Phillips**
Northern Michigan University

We outline the structure of conjugacy closed loops, focusing especially on power associative conjugacy closed loops.

The Sabinin product in loops and quasigroups

Peter Plaumann
University of Erlangen-Nuremberg, Germany and UABJO, Mexico

In a paper published in the year 1939 R. Baer has observed that every left loop $(Q, \circ)$ with a right neutral element 1 corresponds to a transitive, but not necessarily faithful, action of a group

G on the set Q. A bit more precisely, choose for G the left multiplication group of $(Q, \circ)$ and for H the stabilizer of 1 in G. Identifying Q with a transversal of the coset space G/H containing the neutral element of G define on Q a multipication $\diamond$ by $x(yH) = (x \diamond y)H$. Then $(Q, \diamond)$ is a left loop isomorphic to $(Q, \circ)$.

More generally we call a triple (G, H, K) consisting of a group G, a subgroup H of G and a transversal K of H in G a *Baer triple*. The question under which conditions a Baer triple gives us a loop already has been treated by Baer. In 1972 Ł. Sabininin, not knowing the results of Baer, described a way to construct the left multiplication group M of a loop L if the given data are L and the stabilizer H of the neutral element of L in M. Sabinin called the group H the *left associant* of the loop L and the group M the semi-direct product of H and L.

My talk is dedicated to the memory of Lev Sabinin. I will give a survey on his construction and will discuss various aspects of it. Among them one finds the concept of the holomorph of a group and of the knit product (Zappa-Szép product) in groups as well as the work of P. O. Mikheev on covering groups of Moufang loops. The latter topic recently has been presented very clearly by J. Hall in 2010.

Algebraic closure of some generalized convex sets

*Gábor Czédli and Anna Romanowska**
Warsaw University of Technology, Warsaw, Poland

Algebraic convex sets over a principal ideal subdomain R of the ring of real numbers are described as algebras equipped with a set of non-associative and non-commutative binary multiplications. They provide models for spaces with holes. Among the algebraic convex sets, geometric convex sets are described as the intersections of convex subsets of real affine spaces with corresponding affine spaces over R. We will introduce the concept of algebraic closure for geometric convex sets, using certain left quasigroup operations, and examine some of its properties. In particular, the algebraic and topological closures of geometric convex subsets of finite-dimensional affine spaces over R coincide.

Towards a characterization of left quasigroup polynomials of small degree over fields of characteristic 2

*Danilo Gligoroski and Simona Samardjiska**
Norwegian University of Science and Technology, Norway

Permutation polynomials (PPs) defined over fields of characteristic 2 are particularly important because of their broad application in cryptography and coding theory. However, their characterization even for small degrees is still a challenging open problem. After the work of Dickson [1] who characterized completely PPs over any finite field up to degree 5, and PPs of degree 6 for odd characteristic, it was only recently that Li et al. [2] extended the characterization of PP for degree 6 and 7 for fields of characteristic 2.

A natural generalization of the notion of permutation polynomials is that of left quasigroup polynomials (LQPs). A particularly interesting class is that of LQPs that are of algebraic degree 2. Their multivariate representation is known under the name of Multivariate Quadratic Quasigroups (MQQs), and these are the basis of the MQQ public key cryptosystems [3,4].

Following the work of Dickson and Li et al., we take a step towards characterization of LQPs of degree up to 6 defined over $\mathbb{F}_{2^k}$ that are of algebraic degree 2. We investigate different types of bivariate polynomials of degree at most 6, and give some necessary and sufficient conditions for these polynomials to define LQPs.

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A dense family of finite 1-generated left-distributive groupoids

Matthew Smedberg
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The study of nonidempotent 1-generated left-distributive groupoids which are not Laver Tables has been slowed by the chaotic combinatorics of this class. I present a simplification of Drapal's characterization scheme, exhibiting an explicit class $\mathcal{F}$ of finite LD groupoids given by five integer parameters, such that every finite 1-generated LD groupoid can be represented as a homomorphic image of a groupoid from $\mathcal{F}$.

I will also discuss the relevance of this family to a problem of Richard Laver, of removing the large-cardinal hypotheses from the proof that the free 1-generated LD groupoid is residually finite.

Quasigroup actions and approximate symmetry

Jonathan Smith
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Groups owe their importance to their permutation actions, which are the basic model for symmetry. Over the past 15 years, these permutation actions have been generalized to quasigroups, where they provide models for approximate symmetry.

After recalling the basic definition of a quasigroup permutation action, with Markov matrices replacing permutation matrices, attention will focus on a selection of the following topics:

1. Lagrangian properties.

2. Burnside's Lemma.

3. Sylow theory.

4. A simple Bol loop acting on a projective line.

5. Approximately symmetric fractal-type objects.

Topic 3 includes joint work with M. Kinyon and P. Vojtěchovský, topic 4 with K. Johnson, and topic 5 with J. Chalmers.

About isotopy-isomorphy problems for IP-loops

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Theorem. If two IP-loops are isotopic, then they are pseudoisomorphic.

Corollary 1. If two commutative IP-loops are isotopic, then they are isomorphic.

Corollary 2. If two commutative Moufung loops are isotopic, then they are isomorphic.

Irreducible representations of Jordan superalgebras $Kan(n)$

Olmer Folleco Solarte

The classification of irreducible bimodules over simple finite-dimensional Jordan superalgebras over an algebraically closed field of characteristic 0 is practically finished in the articles cited below. The only open case is a structure of irreducible bimodules over the Kantor superalgebras related with Grassmann superalgebras $Kan(n)$ for $n = 2, 3, 4$.

In our work we classify irreducible bimodules over the superalgebra $Kan(n)$ for all n and over a field of characteristic $\neq 2$. The work is a part of the author's PhD thesis done under the direction of Prof. Ivan Shestakov at the Universidade de São Paulo.

Commutator theory for loops I

David Stanovský and Petr Vojtěchovský*
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Using the Freese-McKenzie commutator theory for congruence modular varieties, we develop commutator theory for the variety of loops. The main result is a relation between generators of the commutator of two normal subloops, and generators of the total inner mapping group of a loop.

We argue that some standard definitions of loop theory, drawn upon direct analogy to group theory, should be revised. In particular, we show that Bruck's notion of solvability is strictly weaker than solvability in the sense of commutator theory, and question certain results, such as Glaubermann's extension of the Feit-Thompson odd order theorem to Moufang loops.

Free Steiner loops

*Alexander Grishkov, Marina Rasskazova and Izabella Stuhl**
University of Sao Paulo, Brazil

Free Steiner loops form a variety, therefore we can operate with free objects on this variety. We give a construction of free Steiner loops, determine their multiplication group and focus our attention on automorphisms.

The main results are that all automorphisms of the free Steiner loops are tame. Furthermore, the group of automorphisms cannot be finitely generated when the loop is generated by more than 3 elements. In the case of 3-generated free Steiner loop we specify the (triples of) generators of the automorphism group.

Finally, we present some results about centrally nilpotent Steiner loops of class 2.

The exceptions that prove the rule

Tony Sudbery
University of York, UK

Several types of algebraic structures have classification theorems showing that the simple (in a technical sense) examples of the structure fall into a small number of well-understood infinite families, but with a finite number of exceptions. These exceptions are often associated with the non-associative algebra of octonions, which makes them also understandable in the same terms as the infinite families. I will explore this with particular reference to the case of Lie algebras.

Novel construction of Loday-type algebras

Olga Salazar Díaz, Raúl Velásquez and Luis Alberto Wills-Toro*
Universidad de Antioquia, Colombia

P. Kolesnikov and V. Yu. Voronin proved recently that the algorithm proposed by M.R. Bremner, R. Felipe, and J. Sanchez-Ortega provides the varieties of Loday-type algebras according to the construction (K-P construction) devised by P. Kolesnikov and A.P.Pozhidaev. We provide a further construction that generates varieties equivalent to the K-P construction using bimodules over varieties of algebras. Such a construction gives by itself a direct algorithm. It also enables the study of some structure properties of the varieties.

Keywords: Dialgebras, Leibniz algebras, Nonassociative algebras, Bimodules.

MSC: 17A30, 17A32, 17D99.

Commutator theory for loops II

*David Stanovský and Petr Vojtěchovský**
University of Denver, USA

This is a continuation of the talk by David Stanovský. We will focus on applications of the theory to associators, associator subloops and derived subloops.

A simple approach to n-ary Loday algebras

*Olga Salazar Díaz, Raúl Velásquez and Luis Alberto Wills-Toro**
Universidad Nacional de Colombia, Colombia

The Loday-type n-ary algebras have been introduced recently for specific cases. For generic binary algebras P. Kolesnikov and A.P.Pozhidaev provided a construction (K-P construction), which was followed by an algorithm by M.R. Bremner, R. Felipe, and J. Sanchez-Ortega that implements such a construction, as it was proven later by P. Kolesnikov and V. Yu. Voronin.

In a previous work, we have devised a further construction proved to generate varieties equivalent to the K-P construction using bimodules over varieties of algebras. We extend here this construction to n-ary algebras and provide multiple examples: Jordan and Lie Triple disystems, Casas-Loday-Pirashvili n-ary Leibniz algebras.

Keywords: Dialgebras, Leibniz algebras, n-ary algebras, Bimodules.

MSC: 17A40, 17A42, 17A30, 17A32, 17D99.