
On Loday's parametrized one-relation algebras

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Loday's parametrized one-relation algebras are defined by a bilinear operation satisfying this polynomial identity for some scalars x_σ ($\sigma \in S_3$):

$$(ab)c \equiv \sum_{\sigma \in S_3} x_\sigma a^\sigma (b^\sigma c^\sigma).$$

It is an open problem to classify the cases satisfying condition S: the corresponding operads are isomorphic as S -modules to the associative operad, and hence the free algebras are isomorphic as graded vector spaces to the (non-unital) tensor algebra. Livernet and Loday studied the one-parameter family of solutions containing associative and Poisson algebras. We study some new solutions: the family $(ab)c \equiv \lambda c(ab)$ ($\lambda \neq \pm 1$), a one-parameter family of deformations of the Leibniz operad, and the dual family of deformations of the Zinbiel operad.