Magic squares of Lie groups

Tevian Dray, John Huerta, Joshua Kincaid, Corinne A. Manogue, Aaron Wangberg and Robert A. Wilson*
Oregon State University, USA

The Tits-Freudenthal magic square yields a description of certain real forms of the exceptional Lie algebras in terms of a pair of (possibly split) division algebras. At the group level, the first two rows are well understood, including a geometric understanding of the minimal representations of F_4 and E_6 in terms of the Albert algebra. In the third row, the minimal representation of E_7 consists of Freudenthal triples.

We present here several results at the group level: A complete description of the corresponding "2×2" magic square as $SU(2,\mathbb{K}'\otimes\mathbb{K})$, the use of Cartan decompositions involving all 5 real forms of E_6 to identify chains of real subgroups of the particular real form $SL(3,\mathbb{O})$, and a new description of Freudenthal triples in terms of "cubies", the components of an antisymmetric rank-3 representation of (generalized) symplectic groups, thus providing a unified, geometric interpretation of Freudenthal triples as a single object, and a new description of the minimal representation of E_7.

In future work, we hope to extend this construction to the fourth row, ultimately providing a unified description of the full magic square.