Totally automorphic loops

Michael Kinyon
University of Denver, USA

The multiplication group $\mathrm{Mlt}(Q)$ of a loop Q is the group generated by the left and right multiplications L_x, R_x, $x \in Q$, where $L_x y = R_y x = xy$. The inner mapping group $\mathrm{Inn}(Q)$ is the stabilizer in $\mathrm{Mlt}(Q)$ of the identity element 1. A loop is automorphic if every inner mapping is an automorphism.

The recent work of Stanovský and Vojtěchovský on commutator theory for loops (which they will discuss in Friday's morning session) has drawn attention to a larger permutation group, the *total multiplication group* $\mathrm{TMlt}(Q)$ which is generated by the L_x's, the R_x's and the division maps M_x where $M_x y = y \backslash x$. The *total inner mapping group* $\mathrm{TInn}(Q)$ is the stabilizer in $\mathrm{TMlt}(Q)$ of 1.

Call a loop Q *totally automorphic* if every mapping in $\mathrm{TInn}(Q)$ is an automorphism of Q. It is easy to see that every totally automorphic loop is commutative, but in fact, one can say quite a bit more. After a general introduction to automorphic loops (both for my benefit and the benefit of the two speakers to come after me), I will give a complete characterization of totally automorphic loops. It turns out that the variety of totally automorphic loops coincides with a very familiar variety, and the astute in the audience will probably guess what that variety is by the time I reach the end of the talk.