Sign matrices for frames of 2^n-ons under Smith-Conway and Caley-Dickson multiplication

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There has been a great desire to develop doubling formulas that give better algebraic structures as the dimensions of the algebras so formed increase. Whenever these doubling formulas are applied, several interesting loop and algebraic properties are observed on the structures so formed. The Cayley-Dickson formula is given by $(a,b)(c,d) = (ac - \overline{d}b, da + b\overline{c})$ while the Smith-Conway doubling formula is

$$(a,b)(c,d) = \begin{cases} (ac, \overline{a}d), \text{if b=0;} \\ \left(ac - \overline{\overline{b}d}, b\overline{c} + b\overline{\left(\overline{a}.\overline{b^{-1}d}\right)}\right), \text{if b} \neq 0. \end{cases}$$

A Hadamard matrix of order n is a nXn matrix with entries ± 1 such that $HH^T = nI_n$ where I_n is the identity nXn matrix. It is shown that the sign matrices for the frame multiplication under the Smith-Conway and Cayley-Dickson multiplications are Hadamard matrices. Kronecker products are also introduced, and it is shown that the sign matrices for the quaternion and octonion frames are equivalent to Kronecker products.