Some gradings of nonassociative algebras related to fine gradings of exceptional simple Lie algebras

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Gradings on Lie algebras by various abelian groups arise in the theory of symmetric spaces, Kac-Moody algebras, and color Lie superalgebras. In the 1960s, V. Kac classified all gradings by cyclic groups on finite-dimensional simple Lie algebras over complex numbers. In the past two decades, there has been much progress in the study of gradings on simple Lie algebras by arbitrary groups. In particular, over an algebraically closed field of characteristic zero, fine gradings have been classified on all finite-dimensional simple Lie algebras except E_7 and E_8.

In a recent paper, A. Elduque has shown that, in a sense, a fine grading splits into two independent gradings: a grading by a free abelian group, which is also a grading by a root system, and a fine grading by a finite group on the corresponding coordinate algebra. For example, the $\mathbb{Z}_2^3$-grading on the algebra of octonions that arises from the three iterations of Cayley-Dickson doubling process is "responsible" not only for a fine $\mathbb{Z}_2^3$-grading on G_2 but also for fine gradings on F_4 by $\mathbb{Z} \times \mathbb{Z}_2^3$ and on E_r by $\mathbb{Z}^{r-4} \times \mathbb{Z}_2^3$ ($r = 6, 7, 8$). We have a similar picture for the $\mathbb{Z}_3^3$-grading on the simple exceptional Jordan algebra that can be obtained from the first Tits construction.

In this talk, we will discuss the $\mathbb{Z}_4^3$-grading on the simple structurable algebra of dimension 56, the double of the Jordan algebra of Hermitian matrices of order 4 over quaternions. This grading with the $\mathbb{Z}_2^3$ and $\mathbb{Z}_3^3$-gradings mentioned above can be used to construct almost all known fine gradings on exceptional simple Lie algebras.