The Sabinin product in loops and quasigroups

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In a paper published in the year 1939 R. Baer has observed that every left loop $(Q, \circ)$ with a right neutral element 1 corresponds to a transitive, but not necessarily faithful, action of a group G on the set Q. A bit more precisely, choose for G the left multiplication group of $(Q, \circ)$ and for H the stabilizer of 1 in G. Identifying Q with a transversal of the coset space G/H containing the neutral element of G define on Q a multipication $\diamond$ by $x(yH) = (x \diamond y)H$. Then $(Q, \diamond)$ is a left loop isomorphic to $(Q, \circ)$.

More generally we call a triple (G, H, K) consisting of a group G, a subgroup H of G and a transversal K of H in G a *Baer triple*. The question under which conditions a Baer triple gives us a loop already has been treated by Baer. In 1972 Ł. Sabininin, not knowing the results of Baer, described a way to construct the left multiplication group M of a loop L if the given data are L and the stabilizer H of the neutral element of L in M. Sabinin called the group H the *left associant* of the loop L and the group M the semi-direct product of H and L.

My talk is dedicated to the memory of Lev Sabinin. I will give a survey on his construction and will discuss various aspects of it. Among them one finds the concept of the holomorph of a group and of the knit product (Zappa-Szép product) in groups as well as the work of P. O. Mikheev on covering groups of Moufang loops. The latter topic recently has been presented very clearly by J. Hall in 2010.