Name ______________________________

MATH-1951

Quiz 2 - (2.3 - 2.6)

Answer the following questions, and show your work. Scientific calculator only.

[1] (3 points total) Evaluate the limit, if it exists.

Multiply the numerator and denominator by the conjugate of $\sqrt{x}-6$, which is $\sqrt{x}+6$

$$\lim_{x\to 36}\frac{\sqrt{x}-6}{36-x} = \lim_{x\to 36}\frac{(\sqrt{x}-6)}{(36-x)}\cdot\frac{(\sqrt{x}+6)}{(\sqrt{x}+6)}$$

$$= \lim_{x\to 36}\frac{\sqrt{x}\sqrt{x}+\cancel{6\sqrt{x}}-\cancel{6\sqrt{x}}-6\cdot 6}{(36-x)(\sqrt{x}+6)}$$

$$= \lim_{x\to 36}\frac{x-36}{(36-x)(\sqrt{x}+6)} = \lim_{x\to 36}\frac{-(\cancel{36-x})}{(\cancel{36-x})(\sqrt{x}+6)}$$

$$= \lim_{x\to 36}\frac{-1}{\sqrt{x}+6} = \frac{-1}{\sqrt{36}+6}$$

$$= \frac{-1}{6+6} = \frac{-1}{12}$$

[2] (3 points total) Find the limit.

The highest power of x in the denominator is x, so divide the numerator and denominator by x.

$$\lim_{x\to -\infty}\frac{-1-2x}{3x+5} = \lim_{x\to -\infty}\frac{\frac{-1-2x}{x}}{\frac{3x+5}{x}}$$

$$= \lim_{x\to -\infty}\frac{-\frac{1}{x}-\frac{2\cancel{x}}{\cancel{x}}}{\frac{3\cancel{x}}{\cancel{x}}+\frac{5}{x}}$$

$$= \lim_{x\to -\infty}\frac{\cancel{-\frac{1}{x}}^{\to 0}-2}{3+\cancel{\frac{5}{x}}^{\to 0}} = \frac{-2}{3}$$

[3] (4 points total) For what value of the constant c is the function continuous on $(-\infty, \infty)$? Make sure to show that the function is continuous for this value c.

$$f(x) = \begin{cases} 2(x-c) & , \text{if } x \geq 0 \\ x^2+1 & , \text{if } x < 0 \end{cases}$$

We only need to find c so that f is continuous at $x=0$ since $2(x-c)$ and x^2+1 are continuous on $(-\infty, \infty)$.

So, we find c so that

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = f(0)$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x^2+1 = 0^2+1 = 1$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} 2(x-c) = 2(0-c) = -2c$$

Thus, it must be that $-2c = 1$, which implies $c = \frac{-1}{2}$

Therefore, $\lim_{x \to 0^+} f(x) = -2\left(\frac{-1}{2}\right) = 1 = \lim_{x \to 0^-} f(x)$

Also, $f(0) = 2\left(0 - \frac{-1}{2}\right) = 2\left(\frac{1}{2}\right) = 1$

So, $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 1 = f(0)$