MATH-1951
Quiz 4 - (3.2, 3.3, 3.4)
Answer the following questions, and show your work. Answers need not be simplified.
  Scientific calculator only.

[1] (3 points total) Find the derivative of

$$f(x) = \sin(-x^2)$$

$$f'(x) = \cos(-x^2) \cdot (-x^2)'$$

$$= \cos(-x^2) \cdot (-2x)$$

$$= -2x \cos(-x^2)$$

[2] (3 points total) Find $f'(x)$ and $f''(x)$ of

$$f(x) = 2x^5 e^x$$

Rename $h(x) = 2x^5 e^x$

$f = 2x^5 \qquad g = e^x$

$f' = 10x^4 \qquad g' = e^x$

$$h''(x) = (2x^5 + 10x^4) \cdot (e^x)' + (2x^5 + 10x^4)' \cdot e^x$$

So, $h'(x) = fg' + f'g$

$$h''(x) = (2x^5 + 10x^4) \cdot e^x + (10x^4 + 40x^3) e^x$$

$$= 2x^5 \cdot e^x + 10x^4 \cdot e^x$$

$$\boxed{h'(x) = (2x^5 + 10x^4) \cdot e^x}$$

[3] (2 points total) Differentiate

$$f(x) = \frac{x-1}{x+1}$$

Rename $h(x) = \frac{x-1}{x+1}$

$$h'(x) = \frac{g f' - f g'}{g^2}$$

$f = x - 1 \qquad f' = 1$

$g = x + 1 \qquad g' = 1$

$$= \frac{(x+1) \cdot 1 - (x-1) \cdot 1}{(x+1)^2}$$

$$= \frac{x+1 - (x-1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$h'(x) = \frac{2}{(x+1)^2}$$

[4] (2 points total) For what value(s) of x does the graph of f have a horizontal tangent?

$$f(x) = \frac{x^4 - 6x^3 + 9x^2}{3x}$$

Simplify f first

$$f(x) = \frac{x^4}{3x} - 2x^2 + 3x$$

$$f(x) = \frac{1}{3}x^3 - 2x^2 + 3x$$

Taking derivative $f'(x) = x^2 - 4x + 3$

Horizontal tangent when $f'(x) = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3, 1$ are the values of x for which f has a horizontal tangent