Name _____

MATH-1951
Quiz 5 - (3.5,3.8)
Answer the following questions, and show your work. Answers need not be simplified.
Scientific calculator only.

[1] (5 points total)

a) (4 points) Find dy/dx by implicit differentiation.

$$\sqrt{xy} + y = 1$$

$$\left((xy)^{1/2}\right)' + y' = 0$$

$$\frac{1}{2}(xy)^{-1/2}(xy)' + y' = 0$$

$$\frac{1}{2}(xy)^{-1/2}(y + xy') + y' = 0$$

$$\frac{y}{2\sqrt{xy}} + \frac{x}{2\sqrt{xy}} \cdot y' + y' = 0$$

$$y'\left(\frac{x}{2\sqrt{xy}} + 1\right) = \frac{-y}{2\sqrt{xy}}$$

$$y' = \frac{\frac{-y}{2\sqrt{xy}}}{\left(\frac{x}{2\sqrt{xy}} + 1\right)}$$

b) (1 point) Use your answer to part a) to find the slope of the tangent at $x = 1/2$, $y = 1/2$.

y' at $x = 1/2, y = 1/2$,

$$\frac{\frac{-\frac{1}{2}}{2\sqrt{\frac{1}{2}\cdot\frac{1}{2}}}}{\frac{\frac{1}{2}}{2\sqrt{\frac{1}{2}\cdot\frac{1}{2}}} + 1} = \frac{\frac{-\frac{1}{2}}{1}}{\frac{\frac{1}{2}}{1} + 1} = \frac{-\frac{1}{2}}{\frac{3}{2}}$$

$$= -\frac{1}{2} \cdot \frac{2}{3}$$

$$= -\frac{1}{3}$$

[2] (5 points total) The half-life of cesium-137 is 30 years. Suppose we have 100-mg sample.

a) (2.5 points) Find a formula for the mass remaining after t years.

$P(t) = Ce^{kt}$ $\quad P(30) = 50$ by half-life

$C = 100.$

$50 = 100\, e^{k \cdot 30}$

$\frac{1}{2} = e^{k \cdot 30}$

$\ln \frac{1}{2} = \ln e^{k \cdot 30}$

$\ln \frac{1}{2} = 30k$

$\frac{\ln \frac{1}{2}}{30} = \frac{30k}{30}$

$k = \frac{\ln \frac{1}{2}}{30} = \frac{\ln 1 - \ln 2}{30} = \frac{-\ln 2}{30}$

So, $P(t) = 100\, e^{-\frac{\ln 2}{30} \cdot t}$

b) (1 point) How much of the sample will remain after 100 years? Answer need not be simplified.

$P(100) = 100\, e^{-\frac{\ln 2}{30} \cdot 100} = 100\, e^{\left(-\ln 2 \cdot \frac{10}{3}\right)}$ mg

c) (1.5 points) After how long will only 1 mg remain? Answer need not be simplified.

Solve $P(t) = 1$

$1 = 100\, e^{-\frac{\ln 2}{30} \cdot t}$

$\frac{1}{100} = e^{-\frac{\ln 2}{30} \cdot t}$

$\ln \frac{1}{100} = \ln e^{-\frac{\ln 2}{30} \cdot t}$

$-\ln 100 = \ln 1 \quad -\ln 100 = -\frac{\ln 2}{30} \cdot t$

$t = 30 \cdot \frac{\ln 100}{\ln 2}$ years