Name ______________________________

MATH-1951
Quiz 7 - (4.1, 4.3, 4.5)
Answer the following questions, and show your work. Answers need not be simplified.
Scientific calculator only.

[1] (10 points) Graph the following function, including critical points, regions where the function is increasing or decrasing, points of inflection, regions where the function is concave up or concave down, intercepts where possible, and asymptotes where applicable.

$$f(x) = \frac{9x}{(x-1)^2}$$

$\underline{f'(x):}$ $f(x) = 9\left(\frac{x}{(x-1)^2}\right)$. So, $f'(x) = 9\left(\frac{(x-1)^2 \cdot 1 - x \cdot 2(x-1)}{(x-1)^4}\right)$

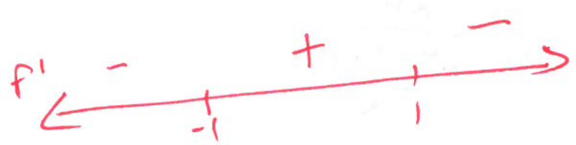
$$= 9 \cdot \cancel{(x-1)}\left(\frac{x-1-2x}{(x-1)^{\cancel{4}3}}\right) = 9\left(\frac{-x-1}{(x-1)^3}\right)$$

$f'(x) = 0$ when $x+1=0 \Rightarrow x=-1$

$$= -9\left(\frac{x+1}{(x-1)^3}\right)$$

$f'(x)$ is undefined when $x = 1$

$$f'(x) = -9\left(\frac{x+1}{(x-1)^3}\right)$$



So, f has $\underline{\text{local min}}$ at $x=-1$

$$f(-1) = \frac{9\cdot -1}{(-1-1)^2} = -\frac{9}{4}$$

$\underline{\text{Increasing:}}$ $(-1, 1)$ $\underline{\text{Decreasing:}}$ $(-\infty, -1) \cup (1, \infty)$.

$\underline{f''(x):}$ $f''(x) = -9\left(\frac{(x-1)^3 \cdot 1 - (x+1)\cdot 3(x-1)^2}{(x-1)^6}\right) = -9 \cdot \cancel{(x-1)^2}\left(\frac{(x-1) - 3(x+1)}{(x-1)^{\cancel{6}4}}\right)$

$$= -9\left(\frac{x-1-3x-3}{(x-1)^4}\right)$$

$$= -9\left(\frac{-2x-4}{(x-1)^4}\right)$$

$$= 18\left(\frac{x+2}{(x-1)^4}\right)$$

$f''(x) = 0$ when $x+2=0 \Rightarrow x=-2$. $f''(x)$ is undefined when $x=1$.

f''	$-$	-2	$+$	1	$+$

So, f has an <u>inflection</u> at $x=-2$.

<u>Concave up:</u> $(-2,\infty)$

<u>Concave down:</u> $(-\infty,-2)$

<u>Intercepts:</u> $f(x)=0$ when $9x=0 \Rightarrow x=0$

$$f(0) = \frac{9\cdot 0}{(0-1)^2} = 0$$

<u>Horizontal asymptotes:</u>

$$\lim_{x\to+\infty} \frac{9x}{(x-1)^2} = \lim_{x\to+\infty} \frac{9x}{x^2-2x+1} = \lim_{x\to+\infty} \frac{\frac{9x}{x^2}}{\frac{x^2-2x+1}{x^2}} = \lim_{x\to+\infty} \frac{\cancel{\frac{9}{x}}\to 0}{1-\cancel{\frac{2}{x}}\to 0+\cancel{\frac{1}{x^2}}\to 0} = 0$$

$$\lim_{x\to-\infty} \frac{9x}{(x-1)^2} = \lim_{x\to-\infty} \frac{\cancel{\frac{9}{x}}\to 0}{1-\cancel{\frac{2}{x}}\to 0+\cancel{\frac{1}{x^2}}\to 0} = 0$$

So, $y=0$ is the horizontal asymptote.

Vertical asymptote: $f(x)$ is undefined at $x = 1$

$\lim_{x \to 1^+} \frac{9x}{(x-1)^2} = +\infty$. $\lim_{x \to 1^-} \frac{9x}{(x-1)^2} = +\infty$. So, $x = 1$ is a vertical asymptote.

Since to the right of $x = 1$, the function is decreasing and concave up, the function exists in the upper right region formed by the asymptotes.

The behavior to the left of $x = 1$ is already determined by the min, intercepts, and inflection.