MATH 1952 Practice Final Exam

Name: _____

Instructions: This test should have 8 problems on 8 pages. Please answer each question as completely as possible, and show all work unless otherwise indicated. You may use an approved calculator for this exam. (Approved: non-graphing, non-programmable, doesn't take derivatives) Answers do not need to be fully simplified unless otherwise specified in the problem.

1. At $t = 0$, a ball is thrown downwards at a speed of 32 feet per second from the lip of the Grand Canyon at a height of 1920 feet above the canyon floor.

(a) Using the fact that acceleration is given by $a(t) = -32$ feet per second squared, find a formula for the velocity $v(t)$ of the ball after t seconds. (Hint: what's $v(0)$?)

(b) Use your answer to (a) to give a formula for the height (or position) $h(t)$ of the ball after t seconds. (Hint: what's $h(0)$?) Use your answer to find out when the ball will hit the canyon floor.

2. Using the pictures below, give the values of the following definite integrals:

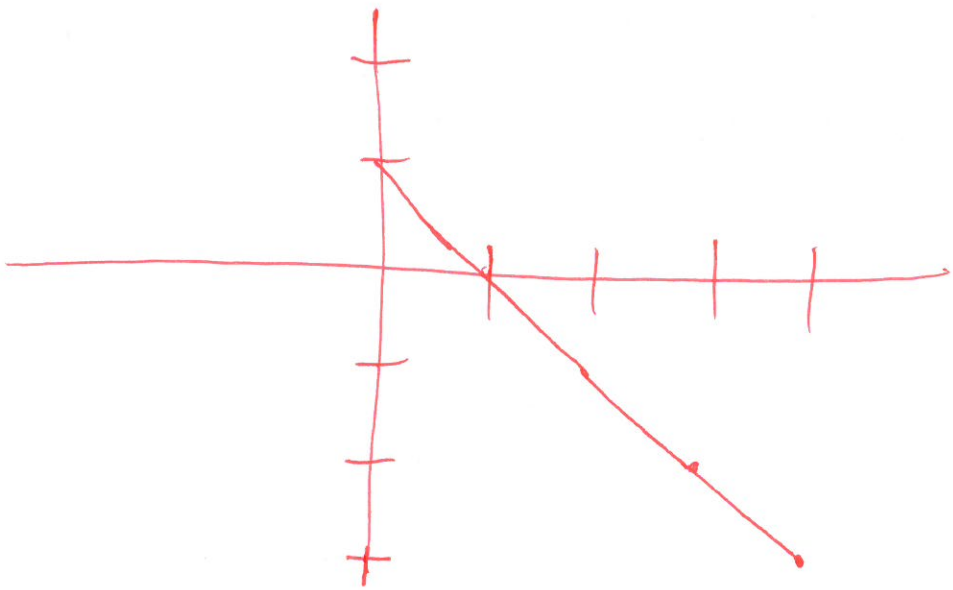
(a) $\int_0^2 f(x)\ dx$

(b) $\int_0^2 g(x)\ dx$

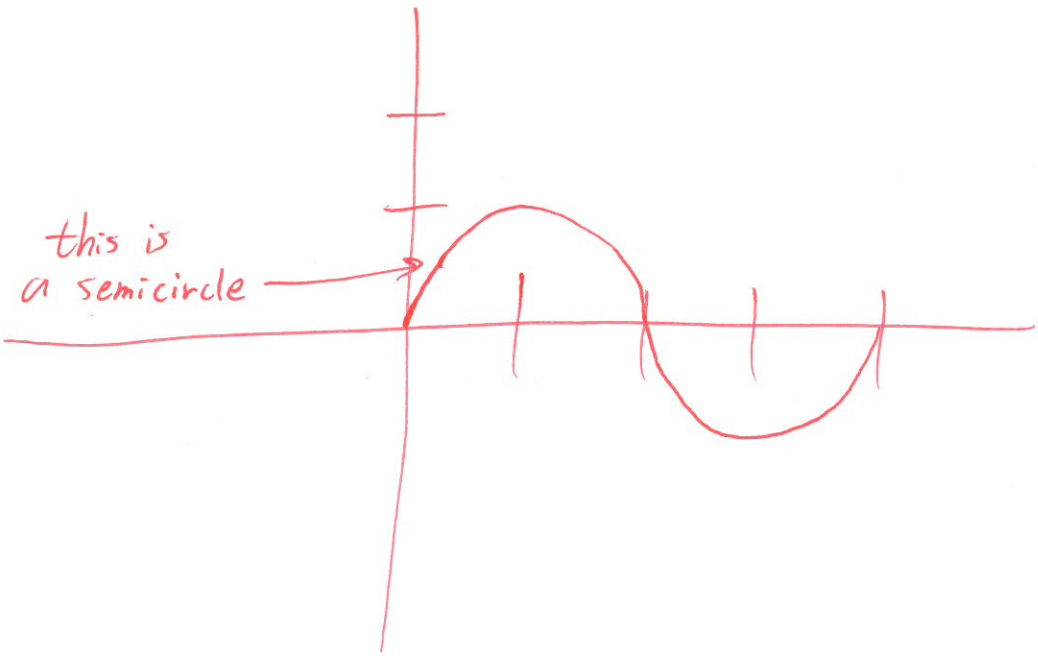
(c) $\int_0^2 2f(x) + 3g(x)\ dx$

(d) $\int_2^4 f(x)\ dx$

$y = f(x)$



$y = g(x)$



3. Find the area between the curves $y = x^2$ and $y = 8 - x^2$.

4. The curve $y = \ln x$ between $x = 1$ and $x = 3$ is rotated around the x-axis, yielding a solid. Find the volume of that solid.

5. The curve $x = \frac{y^2}{8} - \ln y$ between $y = 1$ and $y = 3$ is rotated around the x-axis, yielding a solid. Find the surface area of that solid.

6. Find the average value of the function $f(x) = \frac{\sqrt{x^2-1}}{x^4}$ over the interval from $x = 1$ to $x = \sqrt{2}$.

7. Approximate the definite integral $\int_0^8 \frac{1}{1+x^4}$ by using

(a) Left endpoints with $n = 4$

(b) Midpoints with $n = 4$

(c) Simpson's Rule with $n = 4$.

8. Compute the following indefinite integrals via any method you wish.

(a) $\int \frac{x^2+5}{\sqrt{x}}\, dx$

(b) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$

(c) $\int \frac{4x+6}{x^2+4x+5}\, dx$

Trig identities list:

$\sin^2 x + \cos^2 x = 1$
$\tan^2 x + 1 = \sec^2 x$
$\cot^2 x + 1 = \csc^2 x$
$\sin(2x) = 2 \sin x \cos x$
$\cos(2x) = \cos^2 x - \sin^2 x$
$\sin^2(x) = \frac{1-\cos^2 x}{2}$
$\cos^2(x) = \frac{1+\cos^2 x}{2}$