MATH 1953 Practice Final Exam

Name: ____

Instructions: Please answer each question as completely as possible, and show all work unless otherwise indicated. You do not need to write series in summation notation, but you should write enough terms so that the "pattern" of the terms is obvious. (For instance, $1 + 3x + 5x^2 + 7x^3 + 9x^4 + \ldots$ is OK, but $1 + 3x + \ldots$ is not.) You may use an approved calculator, where "approved" means non-graphing, non-programmable, and cannot take derivatives.

1. Find the area trapped between the x-axis and the curve $x = 5t^3$, $y = 1 - t^2$.

2. Find the length of the portion of the curve $x = \cos^3 t$, $y = \sin^3 t$ traced out between $t = 0$ and $t = \frac{\pi}{2}$.

3. Sketch a graph of the polar curve $r = 1 + \sin\theta$ for $0 \leq \theta \leq 2\pi$.

4. Compute the limit $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x$ by using L'Hospital's Rule.

5. Does the improper integral $\int_0^{\frac{\pi}{2}} \sec^2 x \; dx$ converge or diverge? (Hint: Remember that $\sec x = \frac{1}{\cos x}$. Which is the "bad value" for $\sec^2 x$?)

6. Test the following series for convergence or divergence. You DO NOT need to find the exact sum of the series! Clearly state what series test you apply and explain why the hypotheses are justified.

(a) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n}$

7. (a) Use the definition of Taylor series to give a Taylor series representation of $f(x) = x^{-2}$ with center $x = 1$. Simplify your final answer: it should have no factorials in it (after cancellation).

(b) Use differentiation on part (a) to write the Taylor series representation of $f(x) = x^{-3}$ with center $x = 1$.

8. Use the Taylor series of e^x centered at $a = 0$ to give an approximation of $e^{-0.1}$ to within 0.001. (Hint: this should end up an alternating series!) A calculator-obtained answer with no supporting Taylor series will receive no credit.

9. Find the exact value of the geometric series $\sum_{n=2}^{\infty} \frac{2^n}{5 \cdot 3^n}$.

10. Find the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{n}{(n^3+1)2^n}(x+2)^n$.