Friday Week 4

Calculus III

Let $\{a_n\}$ be the sequence defined by $a_n = \frac{3-n}{4+n}$ for all $n \geq 1$.

(a) Show that $\{a_n\}$ is monotonic by showing that the function $f(x) = \frac{3-x}{4+x}$ is monotonic on $(1, \infty)$.

(b) Show that $\{a_n\}$ is monotonic by comparing a_n and a_{n+1} for any $n \geq 1$.

(c) Is $\{a_n\}$ is bounded above? bounded below? If so, find an upper bound and a lower bound.

For each statement below, indicate if the statement is true or false. If the statement is true, provide an explanation. If false, provide a counterexample.

1. **True or False.** If $a_n = f(n)$ for a function $f(x)$ and $\{a_n\}$ is monotonic, then $f(x)$ is monotonic on $(1, \infty)$.

2. **True or False.** If $\{b_n\}$ diverges and $0 \leq b_n \leq a_n$ for all $n \geq 1$, then $\{a_n\}$ diverges.

3. **True or False.** Every unbounded sequence diverges.

4. **True or False.** If $\{a_n\}$ is decreasing, $\{b_n\}$ is increasing, and $b_n \leq a_n$ for all $n \geq 1$, then $\{a_n\}$ and $\{b_n\}$ both converge.