1. \( \sum_{n=0}^{\infty} \frac{n!(x+2)^n}{e^{2n}} \)

Suppose \( x + 2 \neq 0 \). Then

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{(n+1)!(x+2)^{n+1}}{e^{2(n+1)}} \cdot \frac{e^{2n}}{n!(x+2)^n} \\
= \lim_{n \to \infty} \frac{(n+1)(x+2)}{e^2} \\
= \infty
\]

By the Ratio Test, the series diverges (when \( x + 2 \neq 0 \)). If \( x + 2 = 0 \), i.e. \( x = -2 \), then all terms of the series are 0, so the series converges to 0. Thus the power series converges when \( x = -2 \) and diverges for all other \( x \)-values.

**Center:** -2  
**Radius of convergence:** 0  
**Interval of convergence:** \([-2, -2]\) = \{-2\}

2. \( \sum_{n=0}^{\infty} \frac{(x-2)^n}{5^n \sqrt{n+1}} \)

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{(x-2)^{n+1}}{5^{n+1} \sqrt{n+1} + 1} \cdot \frac{5^n \sqrt{n+1}}{(x-2)^n} \\
= \lim_{n \to \infty} \left| \frac{x-2}{5} \cdot \sqrt{\frac{n+1}{n+2}} \right| \\
= \lim_{n \to \infty} \left| \frac{x-2}{5} \cdot \sqrt{n+1} \cdot \sqrt{\frac{1+\frac{1}{n}}{\frac{1+2}{n}}} \right| \\
= \left| \frac{x-2}{5} \right| \cdot \sqrt{\frac{1+0}{1+0}} \\
= \left| \frac{x-2}{5} \right|
\]

By the Ratio Test, the series converges if

\[ \frac{|x-2|}{5} < 1 \iff |x-2| < 5 \]

\[ \iff -5 < x - 2 < 5 \]

\[ \iff -3 < x < 7 \]
and diverges if
\[
\frac{|x - 2|}{5} > 1 \iff |x - 2| > 5
\]
\[
\iff -(x - 2) > 5 \text{ or } x - 2 < 5
\]
\[
\iff x < -3 \text{ or } x > 7
\]

Hence the power series converges on \((-3, 7)\). Note that we found that the series converges if \(|x - 2| < 5\), which is in the form \(|x - a| < R\), where \(a\) is the center and \(R\) is the radius of convergence. So we can read off that the series is centered at 2 and the radius of convergence is 5. Finally we need to see if the series converges or diverges at each endpoint.

**Left endpoint** \(x = -3\):

The series is
\[
\sum_{n=0}^{\infty} \frac{(-3 - 2)^n}{5^n \sqrt{n + 1}} = \sum_{n=0}^{\infty} \frac{(-5)^n}{5^n \sqrt{n + 1}} = \sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n + 1}},
\]
so we may try the Alternating Series Test. We have that \(\lim_{n \to \infty} \frac{1}{\sqrt{n+1}} = 0\) as required, and \(\frac{1}{\sqrt{n+1}} > \frac{1}{\sqrt{(n+1)+1}}\) for all \(n \geq 0\). Therefore the series is convergent. This shows that the power series converges on \([-3, 7)\), and we are left to check for convergence at the right endpoint.

**Right endpoint** \(x = 7\):

The series is
\[
\sum_{n=0}^{\infty} \frac{(7 - 2)^n}{5^n \sqrt{n + 1}} = \sum_{n=0}^{\infty} \frac{5^n}{5^n \sqrt{n + 1}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n + 1}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}},
\]
which is a divergent \(p\)-series with \(p = \frac{1}{2}\). So the power series does not converge when \(x = 7\), thus the interval of convergence is \([-3, 7)\).

**Center**: 2  
**Radius of convergence**: 5  
**Interval of convergence**: \([-3, 7)\)

3. \(\sum_{n=0}^{\infty} \frac{(x - 1)^n}{n!}\)

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \left| \frac{(x - 1)^{n+1}}{(n + 1)!} \cdot \frac{n!}{(x - 1)^n} \right|
\]
\[
= \lim_{n \to \infty} \left| \frac{x - 1}{n + 1} \right|
\]
\[
= 0
\]

Since the limit is 0 for all values of \(x\), then by the Ratio Test the power series converges for all values of \(x\). Hence the interval of convergence is \((-\infty, \infty) = \mathbb{R}\), centered at 1 with an infinite radius of convergence.
Note: We determine the center of the interval of convergence directly from the power series as it’s presented. A power series is of the form \[ \sum_{n=0}^{\infty} c_n(x - a)^n \] where \( a \) is the center of the interval of convergence, so in this case we have \( c_n = \frac{1}{n!} \), and center \( a = 1 \).

Center: 1

Radius of convergence: \( \infty \)

Interval of convergence: \( (-\infty, \infty) = \mathbb{R} \)