

MATH 1953 Written Homework 4 (due Wednesday, May 1st at the BEGINNING of class!)

Please write solutions to these problems on separate sheet(s) of paper (i.e. don't print and write on this assignment.)

1. For which values of p does the improper integral $\int_e^\infty \frac{1}{x(\ln x)^p} dx$ converge, and what is its value (in terms of p) when it converges? (Show your work!)

2. Use the Comparison Test to show that the improper integral $\int_0^\infty \frac{1}{x^3 + \sqrt{x}} dx$ converges, but DO NOT try to find its value by evaluating the integral (it's not possible!). (HINT: here both endpoints are 'bad', so you'll have to break this integral up. You won't be able to solve this integral exactly, so you'll need to find something for the Comparison Test for each piece of the integral.)

3. Math people are crazy, and insist that the following weird expression, called a **continued fraction**, makes sense:

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}}}$$

To make sense of this, we want to think of it as the limit of a sequence.

(a) Write the first five terms of a sequence by cutting off the continued fraction before each + sign. (For example, the first term is 1, and the second term is $1 + \frac{1}{1}$.)

(b) Simplify your terms from (a) into single fractions. Do you notice a pattern for the numerators and denominators? (It's OK to use Google here!)

(c) Now, use a calculator to write your answers as decimals. Do they seem to be approaching a limit? If so, can you find out what this limit is commonly known as? (It's OK to use Google here, but you'll probably want to use your answer from (b) to get at least 9 terms first!)