1. The infinite series \( \sum_{n=3}^{\infty} \frac{1}{n \ln n} = \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} + \frac{1}{5 \ln 5} + \ldots \) diverges to \( \infty \). (By the Integral Test!) However, it does this EXTREMELY slowly: the sum of the first 10 terms is still only a tiny bit bigger than 1! Let’s figure out how many terms need to be added in order to become larger than 3.

Use a graph of \( y = \frac{1}{x \ln x} \) to explain why the partial sum \( \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} + \ldots + \frac{1}{N \ln N} \) is greater than \( \int_3^{N+1} \frac{1}{x \ln x} \, dx \). Then, use this fact to give a value of \( N \) so that this partial sum is larger than 3. (It will be very large!!)

2. Use the estimation formulas we discussed for the Integral Test to estimate \( \sum_{n=1}^{\infty} \frac{1}{n^3} \) to within 0.001. In fact the value of this series is known to sum to \( \frac{\pi^4}{90} \); check that your answer is indeed within 0.001 of \( \frac{\pi^4}{90} \).

3. Use the Comparison Test to show that \( \sum_{n=2}^{\infty} \frac{1}{n!} \) converges. (In fact, it converges to \( e - 2 \), but we won’t learn why this is until almost the end of the course.)