MATH 1953 Written Homework 6 (due Wednesday, May 15th at the BEGINNING of class!)

Please write solutions to these problems on separate sheet(s) of paper (i.e. don't print and write on this assignment.)

**1.** The infinite series $\sum_{n=3}^{\infty} \frac{1}{n \ln n} = \frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} + \frac{1}{5 \ln 5} + \ldots$ diverges to $\infty$. (By the Integral Test!) However, it does this EXTREMELY slowly; the sum of the first 10 terms is still only a tiny bit bigger than 1! Let's figure out how many terms need to be added in order to become larger than 3.

Use a graph of $y = \frac{1}{x \ln x}$ to explain why the partial sum $\frac{1}{3 \ln 3} + \frac{1}{4 \ln 4} + \ldots + \frac{1}{N \ln N}$ is greater than $\int_3^{N+1} \frac{1}{x \ln x} \, dx$. Then, use this fact to give a value of $N$ so that this partial sum is larger than 3. (It will be very large!!!)

**2.** Use the estimation formulas we discussed for the Integral Test to estimate $\sum_{n=1}^{\infty} \frac{1}{n^4}$ to within 0.001. In fact the value of this series is known to sum to $\frac{\pi^4}{90}$; check that your answer is indeed within 0.001 of $\frac{\pi^4}{90}$.

**3.** Use the Comparison Test to show that $\sum_{n=2}^{\infty} \frac{1}{n!}$ converges. (In fact, it converges to $e - 2$, but we won't learn why this is until almost the end of the course.)