MATH 1953 Written Homework 9 (due Friday, June 7th at the BEGINNING of class!)

Please write solutions to these problems on separate sheet(s) of paper (i.e. don't print and write on this assignment.)

1. One can show that the following is a power series representation of the function $\sqrt{1+x}$:

$$\sum_{n=0}^{\infty} (-1)^n \frac{(2n)!}{(1-2n)4^n (n!)^2} x^n.$$

Find the radius of convergence for this power series (do not attempt to check what happens at the endpoints, it's not really possible with what we know!)

2. (You DO NOT need to find the interval of convergence for any of these, just series!) Starting with the series from problem 1,

(a) Find a power series for $(1+x)^{-1/2}$. (Hint: derivative!)
(b) Find a power series for $(1-x^2)^{-1/2}$.
(c) Find a power series for $\arcsin(x)$.

3. Consider the function $f(x) = \frac{1}{(1+x)^2}$.

(a) Find a formula for $f^{(n)}(0)$ for $n \geq 0$.
(b) Find a formula for the Taylor series for $f(x)$ centered at $x = 0$.
(c) Find a power series expansion for $f(x)$ centered at $x = 0$ in a different way - by modifying the (geometric) power series for $\frac{1}{1-x}$.
(d) Check that your answers to (b) and (c) are the same.

4. Use Taylor/power series formulas to evaluate the following limits.

(a) Use the Taylor/power series for e^x centered at $x = 0$ to find the limit

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}.$$

(b) Use the Taylor/power series for $\ln x$ centered at $x = 1$ to find the limit

$$\lim_{x \to 1} \frac{(\ln x) + 1 - x}{(1-x)^2}.$$