Summary of convergence/divergence tests for infinite series:

- **Geometric series**: The series $\sum_{n=0}^{\infty} r^n$ converges to $\frac{1}{1-r}$ if $-1 < r < 1$ and diverges otherwise.

- **$p$-series**: The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges otherwise.

- **Divergence Test**: If the terms $x_n$ do not approach 0 as $n \to \infty$, then the series $\sum_{n=1}^{\infty} x_n$ diverges.

- **Integral Test**: If the terms $x_n$ are positive and decreasing, and $x_n$ can be “turned into a function” $f(x)$, then the series $\sum_{n=1}^{\infty} x_n$ and the improper integral $\int_{n=1}^{\infty} f(x) \, dx$ have the same convergence status.

- **Comparison Test**: If $0 \leq x_n \leq y_n$ for all $n$, and the new series $\sum_{n=1}^{\infty} y_n$ converges, then the original series $\sum_{n=1}^{\infty} x_n$ converges. If $0 \leq y_n \leq x_n$ for all $n$, and the new series $\sum_{n=1}^{\infty} y_n$ diverges, then the original series $\sum_{n=1}^{\infty} x_n$ diverges. If you get a situation/inequality not listed above, the test was inconclusive and you have to use another one.

- **Limit Comparison Test**: If $\frac{x_n}{y_n} \to L \neq 0$ (and $L$ is a finite number), then the series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ have the same convergence status. If $L = 0$ or $L = \pm\infty$, the test was inconclusive and you have to use another one.

- **Alternating Series Test**: If you have a series $\sum_{n=1}^{\infty} (-1)^n x_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} x_n$, and the terms $x_n$ are positive, approach 0, and are decreasing, then the original series converges.

- **Absolute Convergence Test**: If you have a series $\sum_{n=1}^{\infty} x_n$, and the series $\sum_{n=1}^{\infty} |x_n|$ converges, then the original series $\sum_{n=1}^{\infty} x_n$ absolutely converges, which means that it converges.

- **Root Test**: If $\sqrt[n]{|x_n|} = |x_n|^{1/n}$ approaches a limit $R$, then the series $\sum_{n=1}^{\infty} x_n$ converges if $R < 1$ and the series $\sum_{n=1}^{\infty} x_n$ diverges if $R > 1$. If $R = 1$, this test was inconclusive and you have to use another one.

- **Ratio Test**: If $\left|\frac{x_{n+1}}{x_n}\right|$ approaches a limit $R$, then the series $\sum_{n=1}^{\infty} x_n$ converges if $R < 1$ and the series $\sum_{n=1}^{\infty} x_n$ diverges if $R > 1$. If $R = 1$, this test was inconclusive and you have to use another one.
Approximation formulas:

- **Positive series:** For an infinite series \( \sum_{n=1}^{\infty} f(n) \) with positive and decreasing terms, the error from approximating with the \( N \)th partial sum is less than \( \int_N^{\infty} f(x) \, dx \).

- **Alternating series:** For an infinite series \( \sum_{n=1}^{\infty} (-1)^n x_n \) or \( \sum_{n=1}^{\infty} (-1)^{n+1} x_n \) with \( x_n \) positive and decreasing, the error from approximating with the \( N \)th partial sum is less than \( x_{N+1} \).

Formulas for parametric/polar functions:

- **Parametric slope:** \( \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} \) or \( \frac{y'}{x'} \)

- **Parametric area:** Area = \( \int_{t_1}^{t_2} y \, dx \, dt \) or \( \int_{t_1}^{t_2} y \, x' \, dt \)

- **Parametric arc length:** Length = \( \int_{t_1}^{t_2} \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt \) or \( \int_{t_1}^{t_2} \sqrt{(x')^2 + (y')^2} \, dt \)

- **Polar conversion:** \( y = r \sin \theta, \quad x = r \cos \theta, \quad r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x} \)

- **Polar slope:** \( \frac{dy}{dx} = \frac{(dr/d\theta)}{(d\theta/dr)} \) or \( \frac{(r \sin \theta)'}{(r \cos \theta)'} \)

- **Polar area:** Area = \( \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 \, d\theta \)

- **Polar arc length:** Length = \( \int_{\theta_1}^{\theta_2} \sqrt{r^2 + (dr/d\theta)^2} \, d\theta \)

- **Useful trig formulas:** \( \sin^2 \theta + \cos^2 \theta = 1, \quad \sin^2 \theta = \frac{1 - \cos(2\theta)}{2}, \quad \cos^2 \theta = \frac{1 + \cos(2\theta)}{2} \)

Taylor series:

- **Taylor series formula:** The Taylor series for \( f(x) \) centered at \( x = a \) is \( \sum_{n=0}^{\infty} c_n (x - a)^n \), where \( c_n = \frac{f^{(n)}(a)}{n!} \).

  - \( \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \), interval of convergence \((-\infty, \infty)\)

  - \( \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \), interval of convergence \((-\infty, \infty)\)

  - \( \arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \), interval of convergence \([-1, 1] \)

  - \( e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \), interval of convergence \((-\infty, \infty)\)

  - \( \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n \), interval of convergence \((-1, 1)\)