

11.7: How to decide which test to use to check convergence/divergence?

This is just one way I'm suggesting to attack the question of which test to use; if you have other ways to think about it, that's great! In the end, all that's important is that you choose a test which actually works for the series you're considering.

1. Do the terms even approach 0? If not, then you get to conclude that the series **diverges** because of the **Divergence Test**.
2. If the terms approach 0, then look at the **sign** of the terms of the series.
 - 2a. If the terms can be both positive and negative, but oscillate unpredictably (say because of a trig function, or a weird power of -1 like $(-1)^{n^3+n^6}$), then you should probably use the **Absolute Convergence Test**.
 - 2b. If the terms alternate sign like $- + - + - +$ or $+ - + - + -$, then you can try the **Alternating Series Test**.
3. If the terms are all positive, then there are a bunch of tests that might work, and no perfect way to choose one. Here are some clues:
 - 3a. If you see factorials or something to the n th power, then this is probably a **Ratio/Root Test** problem. Remember that factorials are better for the Ratio Test, and polynomials are usually better for the Root Test. If you don't see something to the n th power or a factorial, **DO NOT** use the Ratio/Root Test; you'll always get $R = 1$, which is inconclusive!
 - 3b. If the series looks "similar" to a simpler series that you can work with (maybe just including some added/subtracted stuff that's unimportant for large n), consider the **Limit Comparison Test**.
 - 3c. If the series looks like something that you can integrate if all n turned into x (often there's a u -sub that you can see easily), then consider the **Integral Test**.
 - 3d. If none of the above work, but you can see some obvious useful inequality to a series you know, then you can try the **Comparison Test**. Just remember that half the time this won't yield a useful result, which is why I'm listing it as a last option.