# 11.7: Summary of our tests for convergence/divergence of an infinite series

These are quick summaries to fit on one sheet; if you aren't sure about anything, consult your book/notes for carefully stated versions.

- **Divergence Test:** If the terms $x_n$ do not approach 0, then the series $\sum_{n=1}^{\infty} x_n$ **diverges**.

- **Integral Test:** If the terms $x_n$ are positive and decreasing, and $x_n$ can be "turned into a function" $f(x)$, then the series $\sum_{n=1}^{\infty} x_n$ and the improper integral $\int_{n=1}^{\infty} f(x)\, dx$ **have the same convergence status**.

- **Comparison Test:** If $0 \leq x_n \leq y_n$ for all $n$, and the new series $\sum_{n=1}^{\infty} y_n$ converges, then the original series $\sum_{n=1}^{\infty} x_n$ **converges**. If $0 \leq y_n \leq x_n$ for all $n$, and the new series $\sum_{n=1}^{\infty} y_n$ diverges, then the original series $\sum_{n=1}^{\infty} x_n$ **diverges**. If you get a situation/inequality not listed above, the test was **inconclusive** and you have to use another one.

- **Limit Comparison Test:** If $\frac{x_n}{y_n} \to L \neq 0$ (and $L$ is a finite number), then the series $\sum_{n=1}^{\infty} x_n$ and $\sum_{n=1}^{\infty} y_n$ **have the same convergence status**. If $L = 0$ or $L = \pm\infty$, the test was **inconclusive** and you have to use another one.

- **Alternating Series Test:** If you have a series $\sum_{n=1}^{\infty} (-1)^n x_n$ or $\sum_{n=1}^{\infty} (-1)^{n+1} x_n$, and the terms $x_n$ are positive, approach 0, and are decreasing, then the original series **converges**.

- **Absolute Convergence Test:** If you have a series $\sum_{n=1}^{\infty} x_n$, and the series $\sum_{n=1}^{\infty} |x_n|$ converges, then the original series $\sum_{n=1}^{\infty} x_n$ **absolutely converges**, which means that it **converges**.

- **Root Test:** If $\sqrt[n]{|x_n|} = |x_n|^{1/n}$ approaches a limit $R$, then the series $\sum_{n=1}^{\infty} x_n$ **converges** if $R < 1$ and the series $\sum_{n=1}^{\infty} x_n$ **diverges** if $R > 1$. If $R = 1$, this test was **inconclusive** and you have to use another one.

- **Ratio Test:** If $\frac{|x_{n+1}|}{|x_n|} = \left|\frac{x_{n+1}}{x_n}\right|$ approaches a limit $R$, then the series $\sum_{n=1}^{\infty} x_n$ **converges** if $R < 1$ and the series $\sum_{n=1}^{\infty} x_n$ **diverges** if $R > 1$. If $R = 1$, this test was **inconclusive** and you have to use another one.