## MATH 3162 Homework Assignment 7

**Instructions:** Solve and turn in all of the assigned problems, showing ALL steps or reasoning used in your solutions.

Due on Monday, March 18th, at the BEGINNING of class.

Abbott: 7.2.3, 7.3.3, 7.3.9

• If f is continuous on [a, b],  $f(x) \ge 0$  for all  $x \in [a, b]$ , and f(x) is not the zero function, prove that  $\int_a^b f(x) \, dx > 0$ .

• Suppose that  $f_n(x)$  is a sequence of integrable functions on [a, b] which is decreasing in TWO ways: for each n,  $f_n(x)$  is a decreasing function, and for each x,  $f_n(x)$  is a decreasing sequence. Also suppose that for every  $x \in (a, b]$ ,  $f_n(x) \to 0$ . Prove that  $\int_a^b f_n(x) dx \to 0$  (as  $n \to \infty$ ).

## Extra problems for graduate students:

Abbott: 7.4.6, 7.4.10

• Prove that  $\int_0^{2\pi} |\sin x|^n dx \to 0$  (as  $n \to \infty$ ). (You DO NOT need to prove that  $|\sin x|^n$  is an integrable function; you may assume this fact.)