MATH 3162 Homework Assignment 7 (part 2)

Instructions: Solve and turn in all of the assigned problems, showing ALL steps or reasoning used in your solutions.

Due on Monday, March 18th, at the BEGINNING of class.

Abbott: 7.4.4 (you may use results from 7.6 to do this problem; in fact I don't know any other way to do it!)

- Prove that a function f defined on $[a, b]$ is ϵ-discontinuous at x (defined in class or on page 240 of your book) if and only if there exist sequences $(y_n), (z_n)$ where $y_n \to x$, $z_n \to x$, and for all $n \in \mathbb{N}$, $|f(y_n) - f(z_n)| \geq \epsilon$.

- Prove that if f and g are continuous functions on $[a, b]$, and for all continuous h on $[a, b]$, $\int_a^b fh = \int_a^b gh$, then $f = g$. (Hint: contrapositive/contradiction!)

- Use the Lebesgue criterion for Riemann integrability to prove that if f, g are integrable functions on $[a, b]$, then fg is also integrable on $[a, b]$.

Extra problems for graduate students:

- Prove that $n \int_0^{2\pi} |\sin x|^n \, dx$ does NOT approach 0 (as $n \to \infty$).