## Topics list for Math 3162 midterm

The midterm will be held in class on Monday, February 18th. Calculators, laptops, notes, cheat sheets, etc. will NOT be allowed. The midterm will cover the material from Sections 4.4-4.5, 5.2-5.4, and 6.2-6.5 of the textbook (not including Abel's Test).

The best ways to prepare for the midterm are to review your class notes, re-read the text material, review the homework problems, and work on other problems in the text. Below is a review of some the main topics from the course.

**4.4:** Know properties of a continuous function f with compact domain K, namely that f is uniformly continuous on K and that the range f(K) is compact (implying that f achieves a max and min on K).

**4.5:** Know the Intermediate Value Theorem and examples of its application.

**5.2:** Know the definition of the derivative and its relationship with difference quotients. Know how to prove basic theorems using this definition (for instance, that (f+g)'(x) = f'(x) + g'(x).) Know how to use information about difference quotients to derive information about a derivative (e.g. the Interior Extremum Theorem.) Know Darboux's theorem and how it can be applied to the derivative even though the derivative may not be continuous (e.g. if f'(x) is never 0, then f'(x) is either always positive or always negative.)

**5.3:** Know the Mean Value Theorem and Generalized Mean Value Theorem, and how they let you use information about the derivative to yield information about difference quotients (e.g. if f'(x) > 0 on (a, b), then f is strictly increasing on (a, b).) Know the 0/0 case of L'Hospital's Rule, and how it can help you compute some limits.

5.4: There's not much testable in this section, but know that this example exists, its properties, and (most importantly) how its continuity is derived from results about uniformly convergent series in Chapter 6.

**6.2:** Know the definition of uniform convergence, and how it differs from pointwise convergence. Know how to verify uniform convergence using the definition, or disprove it using the negation. Know the Cauchy criterion for uniform convergence. Know that the uniform limit of continuous functions is continuous.

**6.3:** Know how uniform convergence relates to differentiability, and how to use uniform convergence of  $f'_n$  to prove differentiability of the limit of  $f_n$  for a sequence  $(f_n)$  of functions.

**6.4:** Know how to apply the results of 6.2 and 6.3 to a series  $\sum_{k=0}^{\infty} f_k(x)$  of functions, by considering (uniform) convergence of the sequence of partial sums  $S_n = \sum_{k=0}^n f_k(x)$ . Know the Term-by-Term Differentiability Theorem and the Weierstrass M-Test.

**6.5:** Know the definition of a power series, and definitions of the interval of convergence and radius of convergence R. Know that R can be written as  $\frac{1}{\limsup \sqrt[n]{|a_n|}}$  (with the conventions that  $\frac{1}{0} = \infty$  and  $\frac{1}{\infty} = 0$ ). Know where you can guarantee uniform convergence of a power series and thereby use results from 6.4 to verify continuity.