MATH 3851 Homework Assignment 1 (due Tuesday, January 16th)

Textbook problems:

- Sec. 2, problem 2(a): Show that for any complex number z, $\mathrm{Re}(iz) = -\mathrm{Im}(z)$.

- Sec. 3, problems 1(a,b): Reduce the quantities (a) $\dfrac{1+2i}{3-4i} + \dfrac{2-i}{5i}$ and (b) $\dfrac{5i}{(1-i)(2-i)(3-i)}$ to real numbers.

- Sec. 5, problem 5(c): Sketch the set of points z in the complex plane which satisfy the inequality $|z - 4i| \geq 4$.

- Sec. 6, problem 1(b): Use properties of the complex conjugate from class to prove that $\overline{iz} = -i\overline{z}$ for any complex number z.

- Sec. 6, problem 9: By factoring $z^4 - 4z^2 + 3$ into two quadratic factors and using the triangle inequality, show the following: if $|z| = 2$, then $\dfrac{1}{z^4 - 4z^2 + 3} \leq \dfrac{1}{3}$.

- Sec. 6, problem 10(b): Prove that z is either real or pure imaginary (pure imaginary means a complex number with real part 0) if and only if $z^2 = \overline{z}^2$.

- Sec. 9, problem 5(d): By using exponential form of complex numbers, show that $(1 + \sqrt{3}i)^{-10} = 2^{-11}(-1 + \sqrt{3}i)$.

- Sec. 9, problem 6: Show that if z_1 and z_2 are complex numbers with $\mathrm{Re}(z_1) > 0$ and $\mathrm{Re}(z_2) > 0$, then $\mathrm{Arg}(z_1 z_2) = \mathrm{Arg}(z_1) + \mathrm{Arg}(z_2)$. (You may use facts we've shown in class about arguments.)

- Sec. 11, problem 4(a): Find all cube roots of -1, graph them in the complex plane as the vertices of a regular polygon, and identify which one is the principal root.

Extra problems:

- Show that $3+i$, 6, and $4+4i$ are the vertices of a right triangle in the complex plane.

- Put the complex number $\sqrt{2}e^{-i\pi/4}$ into rectangular form.