

Practice Midterm Exam for Math 3851

1. Write $(-1 + \sqrt{3}i)^{10}$ in polar form (i.e. $re^{i\theta}$).
2. Write all possible values of $(1 + i)^{1+i}$ in rectangular form (i.e. $x + iy$).
3. At which $z \in \mathbb{C}$ is the function $f(z) = f(x + iy) = (2x^3y - 2xy^3) + i(x^4 + y^4)$ differentiable? At which z is it analytic?
4. Find a function $v(x, y)$ so that the function $f(z) = f(x + iy) = (2x^3y - 2xy^3) + iv(x, y)$ is entire, i.e. analytic throughout \mathbb{C} .
5. Prove the following statement: if $\left| \frac{z+i}{z-i} \right| = 1$, then z is real.
6. Find a branch of the multiple-valued expression $f(z) = \log(i(z + 2i))$ which is analytic for all z in the open disk $D = \{z : |z| < 2\}$.
7. Prove that the following equation is false for all complex numbers: $i \sin z = \cos z$.
8. Sketch each of these sets in the complex plane, and state whether or not each set is (i) open, (ii) closed, (iii) connected, and (iv) bounded. You DO NOT need to explain your answers for this problem.
 - (a) $\{z \in \mathbb{C} : \operatorname{Re}(z) \neq \operatorname{Im}(z)\}$
 - (b) $\{z \in \mathbb{C} : |z| \leq 2\}$