MATH 3851 Homework Assignment 1 (due Wednesday, January 16th)

Textbook problems:

Section 2, problem 2(a): Show that for any complex number z, $\mathrm{Re(iz)} = -\mathrm{Im(z)}$.

Section 3, problems 1(a,b): Reduce the quantities (a) $\frac{1+2i}{3-4i} + \frac{2-i}{5i}$ and (b) $\frac{5i}{(1-i)(2-i)(3-i)}$ to real numbers.

Section 4, problem 5(c): Sketch the set of points in the complex plane which satisfy the inequality $|z - 4i| \geq 4$.

Section 5, problem 1(b): Use properties of the complex conjugate from class to explain why $\overline{iz} = -i\overline{z}$ for any complex number z.

Section 5, problem 9: By factoring $z^4 - 4z^2 + 3$ into two quadratic factors and using the triangle inequality, show the following: if $|z| = 2$, then

$$\left|\frac{1}{z^4 - 4z^2 + 3}\right| \leq \frac{1}{3}.$$

Section 5, problem 10(b): Prove that z is either pure real or pure imaginary (pure imaginary means a complex number with real part 0) if and only if $\overline{z}^2 = z^2$.

Section 8, problem 5(d): By using exponential form of complex numbers, show that $(1 + \sqrt{3}i)^{-10} = 2^{-11}(-1 + \sqrt{3}i)$.

Section 8, problem 6: Show that if z_1 and z_2 are complex numbers with $\mathrm{Re(z_1)} > 0$ and $\mathrm{Re(z_2)} > 0$, then $\mathrm{Arg(z_1z_2)} = \mathrm{Arg(z_1)}+\mathrm{Arg(z_2)}$. (You may use facts we've shown in class about arguments.)

Section 10, problem 3(a): Find all cube roots of -1, graph them in the complex plane as the vertices of a regular polygon, and identify which one is the principal root.

Extra problems:

• Show that $3+i$, 6, and $4+4i$ are the vertices of a right triangle in the complex plane.
• Put the complex number $\sqrt{2}e^{-i\frac{\pi}{4}}$ into rectangular form.