MATH 3851 Homework Assignment 2 (due Wednesday, January 23rd)

Textbook problems:

Section 11, problem 5: If we define the set $S = \{z \,:\, |z| < 1\} \cup \{z \,:\, |z-2| < 1\}$, explain why S is NOT connected.

Section 12, problem 2: Write the function $f(z) = z^3 + z + 1$ in the form $f(z) = u(x, y) + iv(x, y)$. (Here, as usual, x and y represent the real and imaginary parts of z, so $z = x + iy$.)

Section 12, problem 4: Write the function $f(z) = z + \frac{1}{z}$ in the form $f(z) = u(r, \theta) + iv(r, \theta)$. (Here, as usual, r and θ represent the modulus and argument of z, so $z = re^{i\theta}$.)

Section 18, problem 1(b): Use the ϵ-δ definition of limit to prove that $\lim_{z \to z_0} \overline{z} = \overline{z_0}$ for any $z_0 \in \mathbb{C}$. (It may be easier to start with the easier limit question in the "Extra Problems.")

Section 18, problem 5: Show that the limit of the function $f(z) = \left(\frac{z}{\overline{z}}\right)^2$ as z approaches 0 does not exist. Hint: consider points of the form $z = x$ and $z = x + xi$ approaching the origin.

Extra problems:

- For the function $f(z) = 5z - i$, show that $\lim_{z \to 3} f(z) = 15 - i$.

- Show that $\overline{e^z} = e^{\overline{z}}$ for any complex number z.

- For each of the following sets, say whether or not they are (i) open, (ii) closed, (iii) connected, (iv) bounded. If you claim the set does not have some of these properties, explain why. (For instance, if you're saying the set isn't open, give an example of an interior point which isn't in the set. If you're saying that the set isn't bounded, explain why it doesn't lie inside any circle.)

(a) $\{z \,:\, |2z + 3| > 4\}$
(b) $\{z \,:\, \mathrm{Im(z)} = 1\}$
(c) $\{z \,:\, 0 \leq \arg(\mathrm{z}) < \frac{\pi}{4}\}$

- Suppose that U and V are connected sets, and that there is a complex number z which is in both U and V. Explain why $U \cup V$ must be connected.