MATH 4280 Homework Assignment 2

Due on Wednesday, September 27th, at the BEGINNING of class.

Section 1.4: 17, 18(a,b,c), 23

Non-book problem: Suppose that μ is a measure on $\mathcal{B}(\mathbb{R})$ induced by a pre-measure μ_0, where $\mu_0((a, b]) = F(b) - F(a)$ for some increasing, right-continuous function F (as we started discussing at the end of Wednesday's class.)

Prove that μ is shift-invariant (i.e. $\mu(A) = \mu(A + x)$ for every $A \in \mathcal{B}(\mathbb{R})$ and $x \in \mathbb{R}$) if and only if $F = ax + b$ for some $a, b \in \mathbb{R}$.

(In other words, the only shift-invariant measures of this sort are just constant multiples of Lebesgue measure.)