MATH 4280 Homework Assignment 3

Due on Wednesday, October 4th, at the BEGINNING of class.

Section 1.5: 29, 30, 31, 32

- For problem 29, when they say "the nonmeasurable set defined in 1.1," you can use the one we defined the first day of class instead, they're slightly different. The key properties we proved about that set are that the sets $N + q$, where q ranges over $\mathbb{Q} \cap [-1, 1]$, are disjoint, and that their union satisfies

$$[0, 1] \subseteq \bigcup_{q \in \mathbb{Q} \cap [-1,1]} (N + q) \subseteq [-1, 2].$$

- For problem 31, here's a hint: $x \in E - E \iff E \cap (E - x) \neq \varnothing$.

Written problem: If $\mathcal{M}$ is a σ-algebra on a set X and for every n, $f_n : X \to \mathbb{R}$ is a measurable function (where $\mathbb{R}$ is endowed with σ-algebra $\mathcal{B}(\mathbb{R})$), prove that

$$\{x \in X \ : \ f_n(x) \to 0\}$$

is in $\mathcal{M}$, i.e. a measurable subset of X.

- I know that we haven't started Section 2.1 yet, but this problem requires only the definition of a measurable function, which I'd like you to take a look at anyway. The definition is: if $\mathcal{M}$ is a σ-algebra on X and $\mathcal{N}$ is a σ-algebra on Y, then $f : X \to Y$ is measurable if for all $A \in \mathcal{N}$, $f^{-1}(A) \in \mathcal{M}$. In other words, the preimage of measurable sets in the codomain must be measurable in the domain. (Replace "measurable" with "open" and you would have the definition of a continuous function, so there's a nice analogy between measurable and continuous functions.) So for this problem, you know $f^{-1}(S)$ is in $\mathcal{M}$ for all $S \in \mathcal{B}(\mathbb{R})$; the key is then to somehow use that to show that the set in question is also in $\mathcal{M}$.