

## MATH 4290 Homework Assignment 1

Due on Thursday, September 20th, at the BEGINNING of class.

- If  $(X, T)$  is a non-invertible dynamical system and  $x \in X$  is recurrent, prove that  $Tx$  is recurrent.
- If  $(X, T)$  is a non-invertible dynamical system,  $x \in X$  is recurrent, and  $U$  is an open set containing  $x$ , prove that  $R_U(x) := \{n \in \mathbb{N} : T^n x \in U\}$  is infinite.
- If  $(X, T)$  is a minimal dynamical system and  $(Y, S)$  is conjugate to  $(X, T)$ , prove that  $(Y, S)$  is also minimal.
- Prove that there exists  $n \in \mathbb{N}$  so that the decimal expansion of  $2^n$  begins with 777, and give a provable upper bound on  $n$ .
- Define  $(X, T)$  by  $X = \mathbb{T}^2$ , which you can think of as  $[0, 1)^2$  with the top/bottom edges identified and left/right edges identified, and  $T : (x, y) \mapsto (x + \alpha, y + \alpha)$  for  $\alpha \notin \mathbb{Q}$ . Describe, with proof, all minimal subsystems of  $(X, T)$ .
- If  $(X, T)$  is a dynamical system and  $(M, T)$  is a nonempty subsystem, prove that  $(M, T)$  is the unique minimal subsystem of  $(X, T)$  if and only if every subsystem of  $(X, T)$  contains  $(M, T)$ .