MATH 4290 Homework Assignment 4

Due on Thursday, October 11th, at the BEGINNING of class.

• Prove the Hedlund-Morse theorem for two-sided shifts, namely

(a) If $x \in A^{\mathbb{Z}}$ satisfies $c_n(x) \leq n$ for some n, then x is periodic
(b) If $X \subseteq A^{\mathbb{Z}}$ satisfies $c_n(X) \leq n$ for some n, then X is finite

• Design a subshift $X \subseteq A^{\mathbb{N}}$ which satisfies $c_n(X) = n + 2$ for all $n \in \mathbb{N}$. (We could call such a subshift "almost Sturmian.")

• Prove that for every n, the higher-block code ϕ_n defined in class is a conjugacy from the full shift $A^{\mathbb{Z}}$ to its image. (Reminder: for any $x \in A^{\mathbb{Z}}$, $\phi_n(x) \in (A^n)^{\mathbb{Z}}$ is defined by $(\phi_n(x))_j$, the jth letter of $\phi_n(x)$, is equal to $x_j \ldots x_{j+n-1} \in A^n$ for all $j \in \mathbb{Z}$.)

• Define $X \subseteq \{0,1\}^{\mathbb{Z}}$ to be the set of all 0-1 biinfinite sequences such that the number of 0s between any two closest 1 symbols is even. For instance,

$$\ldots 01001000011001 \ldots \in X, \ldots 010010001001 \notin X.$$

Prove that X is NOT a subshift of finite type, i.e. there does not exist a finite set $\mathcal{F}$ of words so that $X = X(\mathcal{F})$.