MATH 4290 Homework Assignment 6

Due on Thursday, November 1st, at the BEGINNING of class.

- Define S to be the shift with alphabet $\{0, 1\}$ consisting of all bi-infinite sequences in which the number of 0s between any closest 1s is even, and the number of 1s between any closest 0s is a multiple of 3. Prove that S is sofic and mixing, and find $h(S, \sigma)$ by using techniques similar to those used to treat the even shift in class.

- Define $A = \{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\}$, $X = A^{\mathbb{N}}$, and the metric d on X by

$$d((x_n), (y_n)) = \sum_{n \in \mathbb{N}} 2^{-n} |x_n - y_n|,$$

where $|x_n - y_n|$ is defined by the usual absolute value in $\mathbb{R}$. (You do NOT have to show that d is a metric.) If σ is the usual left shift on X, prove that $h(X, \sigma) = \infty$.

- Prove that if X is a mixing 1-step SFT (i.e. the forbidden list is given by pairs of adjacent letters) and $|X| > 1$, then $h(X, \sigma) > 0$.

- Define a directed graph G so that the associated edge shift $X(G)$ is mixing and has $h(X(G), \sigma) = \log(\sqrt[3]{2} + 1)$.

- Prove that it is impossible to find G as in the previous problem with $h(X(G)), \sigma) = \log(3 - \sqrt{2})$.