• Show that if \((X, T)\) and \((Y, S)\) are topologically conjugate/isomorphic topological dynamical systems, and if \((X, T)\) is expansive, then \((Y, S)\) is also expansive. (Here, use the definition of expansivity for invertible systems: \((X, T)\) is expansive if there exists a \(\delta > 0\) such that for every \(x \neq y \in X\), there exists \(n \in \mathbb{Z}\) s.t. \(d(T^n x, T^n y) > \delta\).)

• For an invertible topological dynamical system \((X, T)\) with \(X\) finite and endowed with the discrete topology, show that there is a unique \(T\)-invariant Borel probability measure on \((X, T)\) if and only if \((X, T)\) consists of a single periodic orbit. (Note: here a Borel measure just means a measure defined on the entire power set \(P(X)\).)

• Show that if a topological dynamical system \((X, T)\) has two \(T\)-invariant Borel probability measures, then it has uncountably many.

• If \((X, T)\) is an invertible minimal topological dynamical system and \(\mu\) is a \(T\)-invariant probability Borel measure on \(X\), show that for every nonempty open set \(U\), \(\mu(U) > 0\).

• Suppose that \((X, T)\) is an invertible expansive topological dynamical system. I want you to construct a factor map from some two-sided symbolic system \((Y, \sigma)\) to \((X, T)\), where \(Y \subseteq \{1, 2, \ldots, N\}^\mathbb{Z}\). Here is an outline:

  (a) Prove that for the expansiveness constant \(\delta > 0\) of \((X, T)\), if \(x, y\) have the property that \(d(T^n x, T^n y) < \delta\) for all \(n \in \mathbb{Z}\), then \(x = y\).

  (b) Use this to cover \(X\) with a finite collection of closed balls \(A_i\) so that, for any \(x\), knowledge of a sequence \((k_n)_{n \in \mathbb{Z}}\) s.t. \(T^{k_n} x \in A_k\) for all \(n \in \mathbb{Z}\) uniquely determines \(x\).

  (c) Use this to construct a symbolic system \(Y \subseteq \{1, 2, \ldots, N\}^\mathbb{Z}\) and a factor map from \((Y, \sigma)\) to \((X, T)\). Remember that you must show that \(Y\) is closed in the product topology and \(\sigma\)-invariant!