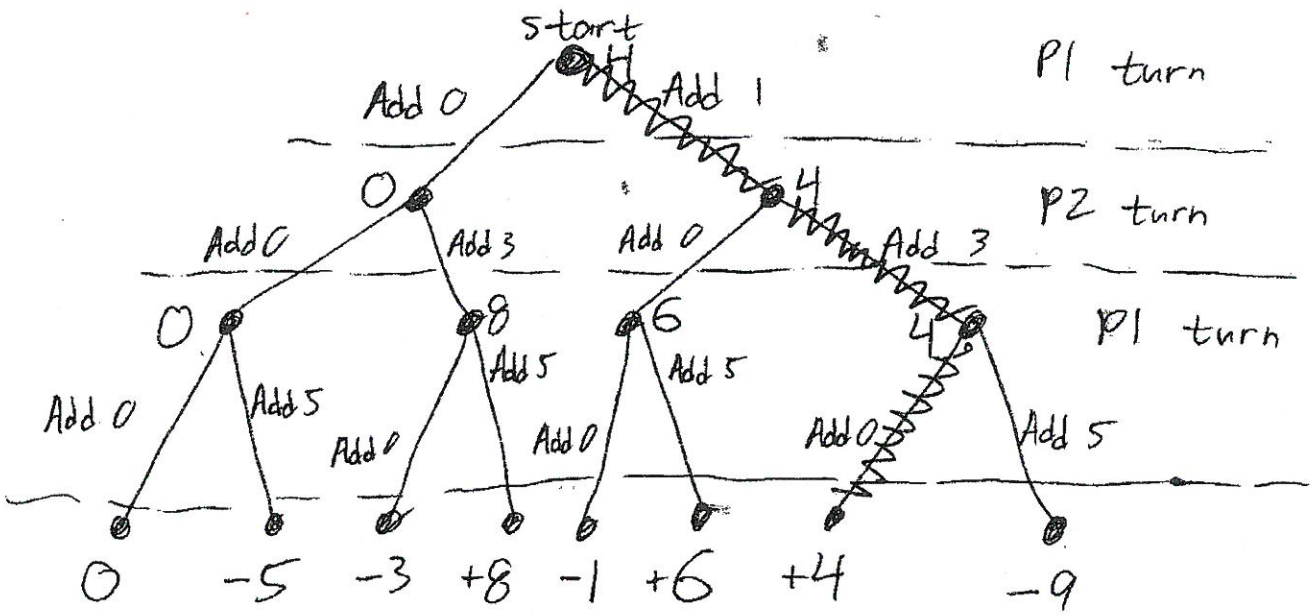
FSEM

[redacted] Assignment 6 Solutions

1. Two players play a game where they keep a running total of numbers, which starts at 0. First, player 1 has the option of adding 1 to the total, or leaving it alone. Then, player 2 has the option of adding 3 to the total, or leaving it alone. Finally, player 1 has the option of adding 5 to the total, or leaving it alone. In the end, if the total is even, then player 1 is paid that many dollars from player 2, and if the total is odd, then player 2 is paid that many dollars from player 1.

Here is a sample play of the game: player 1 adds 1, player 2 adds 3, and finally player 1 adds 5. Then the total is 9, which is odd, so player 2 wins 9 dollars from player 1, resulting in a payoff of −9 to player 1. (Since this is a zero-sum game, as usual we record only the payoffs to player 1.)

Draw the game tree for this game with payoffs for every possible ending, and work backwards to decide what the outcome of the game will be if both players play optimally.



If both players play optimally, final payoff is +4 to player 1: Player 1 adds 1, Player 2 adds 3, Player 1 adds 0.

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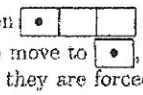
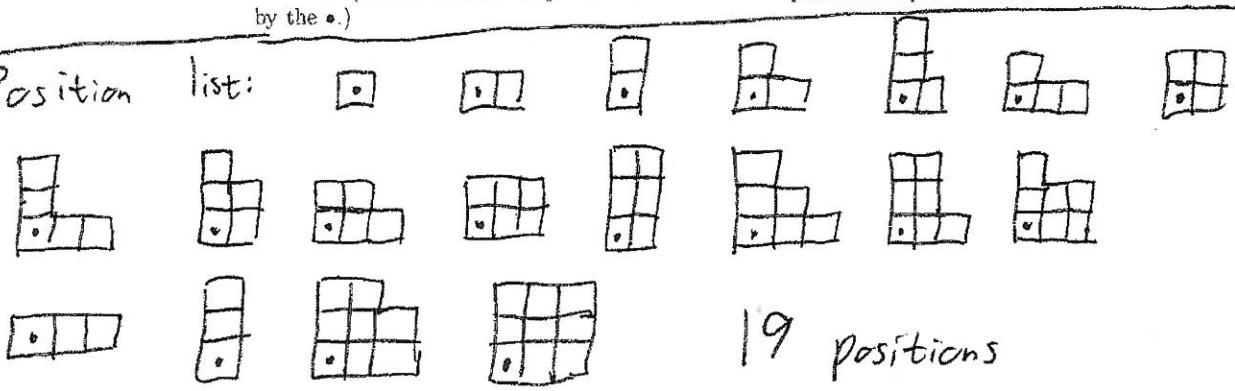
2. Recall that the game of Chomp is played on a rectangular board broken into squares (thought of as a chocolate bar). A turn consists of "eating" any square left in the bar, along with all squares above and to the right of it. The bottom-left square of the bar is "poisoned," (and labeled by a •) and the player to eat it loses the game.

Chomp is impartial, meaning that the possible moves from any given position do NOT depend on which player's turn it is. As we discussed in class, such games can be analyzed by labeling positions as W(inning) or L(osing) for the player who is about to move. Decide which player can force a win in 3×3 Chomp by listing all possible positions in a game of 3×3 Chomp and deciding whether they are W(ins) or L(osses). (HINT: there are 19 positions in 3×3 Chomp.)

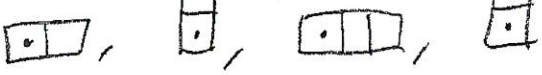
As an example, the position  is a W(in) for the player about to move because a legal move is to move to  , which is a L(oss) for the player about to move. (This is because they are forced to eat the "poisoned" square labeled by the •.)

Position list:



19 positions

 is a L.

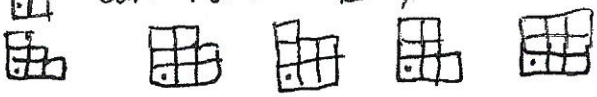
 can all reach  , so are W's.

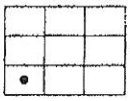
 can only reach  ,  , which are W's, so  is L.

 can all reach  , so are all W's.

 can only reach W's, so are all L's.

 can reach  , so it's a W.  can reach  , so it's W.

 can all reach  so all are W's.

So, we showed that the starting position  of 3×3 Chomp is a W(in) for the first player.