

FSEM Homework Assignment 7

Show your work!

1. The game of Empty and Divide is played between two players. The game starts with two piles of pebbles. The players alternate moves, and each move consists of throwing away one of the piles and then dividing the contents of the other pile into two piles (each of which has at least one pebble). For instance, if the boxes had 11 and 8 pebbles in them, a legal move would be to throw away the pile with 11 pebbles and split the other pile of 8 pebbles into piles of 2 and 6. The game ends when no legal moves can be made, which happens when there is 1 pebble in each pile. As usual, the first player who cannot make a legal move loses.

This game is impartial, and so it is possible to analyze whether game positions are W(ins) (for the player about to move) or L(osses) (for the player about to move) without drawing a tree. For instance, having a pile with 1 pebble and a pile with 2 pebbles is a W, because the player about to move has to split the 2-pebble pile into 1 and 1, and the next player can't move. Having a pile with 1 pebble and a pile with 3 pebbles is a L for the player about to move, because the only legal move is to split the 3-pebble pile into 1 and 2, and we just decided that was a W position for the next player.

Write down all positions for this game with two piles containing up to 7 pebbles, and decide whether they are W or L positions for the player about to move. Make a guess about the general form of all W and L positions, and check that your guess is correct via the methods discussed in class. HINT: usually the end of a Nim-style game is when both piles have 0 chips, but notice that in this game the end happens when both piles have 1 chip!

2. (More fun problem!) We will eventually be playing a game called Duel with our dart guns in class, which involves attempting to hit the other player with a dart at various distances. We'll be playing outside, on a sidewalk with slabs 5 feet long. For this reason, you'll always be taking a shot at your opponent from a distance which is a multiple of 5 feet. A key component of our analysis in class involves knowing a player's probability of hitting their opponent at various distances. For this reason, I'd like you each to compile some data about your accuracy.

For each of the following distances: 15, 20, 25, 30, 35, 40, 45, and 50 feet (eight different distances overall), perform the following experiment. Have a willing participant (friend/classmate/roommate/any other person) stand at the proper distance from you, and shoot a dart at them 10 times, recording your number of hits and then computing your probability to hit at that distance. The opponent is NOT allowed to run/dodge/etc., they should stand perfectly still. Please use common sense; for instance if the opponent thinks they could be hurt by the

dart (say it could hit them in the face), they can dodge and you can redo that trial.

Your final answer should look something like this (with different numbers of course:) 90% hits at 15 feet, 80% hits at 20 feet, 60% hits at 25 feet, etc. Please include your records of misses/makes at each distance, as this will be your “work” for this assignment. Please contact me if you have any questions about what to do.