Introduction to Artificial Intelligence
COMP 3501 / COMP 4704-4
Lecture 5: CSP

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Constraint Satisfaction Problems (CSP)

- CSP's are our first work on factored representations
  - State is no longer a black box
  - State has variables that we can reason about
- Many different problems can be represented as CSP
  - Solved with a single solver

CSP Definition

- CSP has:
  - Set of variables $\{X_1...X_n\}$
  - Set of domains $\{D_1...D_n\}$
  - Set of constraints $\langle scope, relation \rangle$
    - Scope is the variables $\{X_1, X_2\}$
    - Relation can vary; eg $\neq$
- Goal is to find a complete, consistent assignment of variables to values

SAT vs. CSP

- SAT and CSP have the same representative power
  - SAT is a sparser representation
  - CSP has rich constraints

Sample: Map Coloring

- Variables?
- Domains?
- Constraints?

CSP

- Variables:
  - {A, B, C, D, E, F, G}
- Domains:
  - {Red, Green, Blue}
- Constraints
  - ⟨{A, B}, ≠⟩, ⟨{B, C}, ≠⟩, ⟨{C, D}, ≠⟩, ⟨{D, E}, ≠⟩
  - ⟨{A, F}, ≠⟩, ⟨{B, F}, ≠⟩, ⟨{C, F}, ≠⟩, ⟨{D, F}, ≠⟩, ⟨{E, F}, ≠⟩

Types of constraints

- Unary constraints: restrict a single variable
- Binary constraint: restricts two variables / graph edge
- Global constraint: restricts more variables

Example: Cryptarithmetic

- Require that all letters represent different numbers
  - Alldiff constraint

```
  SEND    DONALD     FIFTY    BASE
+ MORE  + GERALD  + STATES  + BALL
 -----    ------  --------  ------
 MONEY    ROBERT   AMERICA   GAMES
```

Solving CSPs

- Two possibilities
  - A mix of both are often used
  - Constraint inference
    - Propagate constraints without explicit search
  - Backtracking search (DFS)

Constraint Inference

- Node consistency
  - Use unary constraints to remove variables from the domain of a node
- Arc consistency
  - Use the constraints on an edge to limit the domain of nodes connected by the edge
  - Compares two variables

Arc Consistency Example

- Works on simple problems
- Doesn't work when trying to 2-color Australia

Constraint Inference

- Path Consistency
  - Compares three variables and two constraints
  - For each assignment of two variables, ensure that the third variable has legal values (reduce to legal values)
  - Will detect that Australia can't be 2-colored
- k-Consistency
  - Look at any *k-1* variable and make sure the *k*th is consistent

Global Constraints

- Different global constraints can be defined with special propagation rules
  - Alldiff - If there are *m* variables and *n* distinct values
    - If $m > n$, then alldiff cannot be satisfied

Sudoku

- What is Sudoku
- How should Sudoku be encoded?

6			1		8	2		3
	2			4			9	
8		3			5	4		
5		4	6		7			9
	3						5	
7			8		3	1		2
		1	7			9		6
	8			3			2	
3		2	9		4			5

			1			7	4	
	5			9			3	2
		6	7			9		
4			8					
	2						1	
					9			5
		4			7	3		
7	3			2			6	
	6	5			4			

Backtracking Search

- DFS in which each node in the tree corresponds to the assignment of a value to a variable
  - Fix the variable chosen at each node in the tree
  - The order we choose variables can make a big difference
  - What if we choose the wrong order for Australia?

Variable & Value ordering

- Choose the most constrained variable first
  - If a variable only has 1 value, there is no choice
  - If one variable has 10 values and another has 2
    - Choose the one with 2 values, as it has tighter constraints to be met
- Choose the least constraining value first
  - If possible, avoid constraining the remaining solution
  - Doesn't matter if we want all solutions

Forward checking

- Each variable assignment means new inference can take place
- MAC algorithm (Maintaining Arc Consistency)
  - Re-performs arc consistency
  - Instead of all arcs, only arcs for neighbors

Backtracking

- Simple approach:
  - Backtrack to last variable assignment
- Better approach:
  - Backtrack to last conflicting variable assignment

No-goods

- We can learn new constraints during search
  - Find combinations of variables that cause failure
  - Store them in a “no-good”

Local search

- An alternate approach is local search
  - Select a conflicting variable
  - Choose the value which minimizes constraints
    - *min-conflicts* heuristic
  - Tabu search: avoid recent states
- Already analyzed in 8-queen puzzle
  - How to represent 8-queens as a CSP?

Next Homework: 7.2