Introduction to Artificial Intelligence
COMP 3501 / COMP 4704-4
Lecture 6: Intro to Propositional Logic

Prof. Nathan Sturtevant



## Lecture Overview [7.1-7.4]

- Logical Agents
- Wumpus World
- Propositional Logic
- Inference
- Theorem Proving via model checking



## Motivation

- Existing techniques help us solve:
  - Shortest path problems
  - Some classes of optimization problems
- What about problems that require logical reasoning?
  - eg creating a Sherlock Holmes agent
    - “When you have eliminated the impossible, whatever remains, however improbable, must be the truth.”



## Logical Agents

- Maintain representation of knowledge of the world
- Use factored state representation
  - States are assignments of values to variables
- Like CSPs can generalize to many different problems
- Can also generalize to different goals

## Knowledge-based agents

- Logical agents maintain world knowledge
  - Knowledge base (KB)
- Knowledge stored in *sentences*
  - Each sentence represents knowledge about the world
    - Sherlock Holmes was a fictional detective



## Knowledge-based agents

- Add knowledge: TELL
- Query knowledge: ASK

- Agent loop:
  - TELL KB about perceptions
  - ASK actions to perform
- ASK not necessarily formulated explicitly



## Knowledge

- Declarative:
  - TELL an agent what is needed
  - No extra knowledge
- Procedural:
  - Encode knowledge in program code
- SAS is often procedural
- Generalized planning is declarative



## Wumpus World

- Performance
  - 1000 for getting gold and returning to start
  - -1000 for dying
  - -10 for shooting the arrow
  - -1 for each action

## Wumpus World

- Environment
  - 4x4 grid of rooms
  - Agent has heading
  - Agent starts at [1, 1]
  - Gold & wumpus randomly placed
  - Probability 0.2 of a pit



## Wumpus World

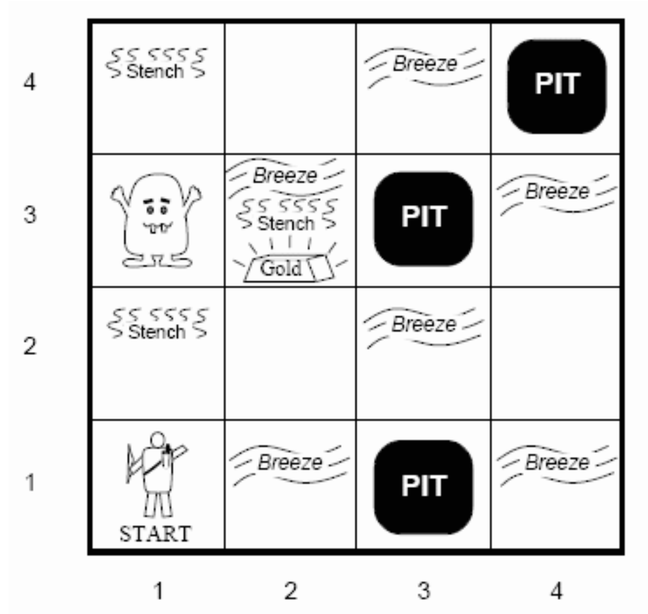
- Actuators
  - Turn right
  - Turn left
  - Forward
  - Shoot
  - Grab
  - Exit



## Wumpus World (WW)

- Sensors
  - Can perceive *stench* from location adjoining (vertically/horizontally) a wumpus
  - Can perceive *breeze* from location adjoining a pit
  - Can perceive *glitter* in cell with gold
  - Can perceive *scream* when wumpus dies





## Logic

- Syntax: defines well-formed sentences
- Semantics: what sentences mean
  - x + y = 4 is true when x = 2 and y = 2
- Model: possible world
  - Includes all assignments of values to x/y
  - If α is true in m: m satisfies α
  - M(α) is the set of all models of α
- What models exist for WW problem?



## Entailment

- α entails ß *or* α ⊨ ß
  - ß follows logically from α
  - In every model in which α is true, ß is also true
    - M(α) ⊆ M(ß)

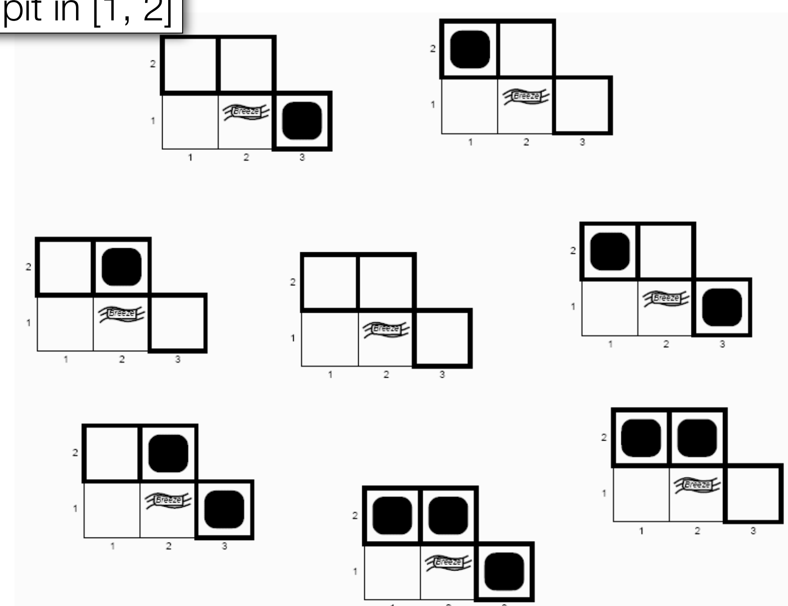


## Entailment examples

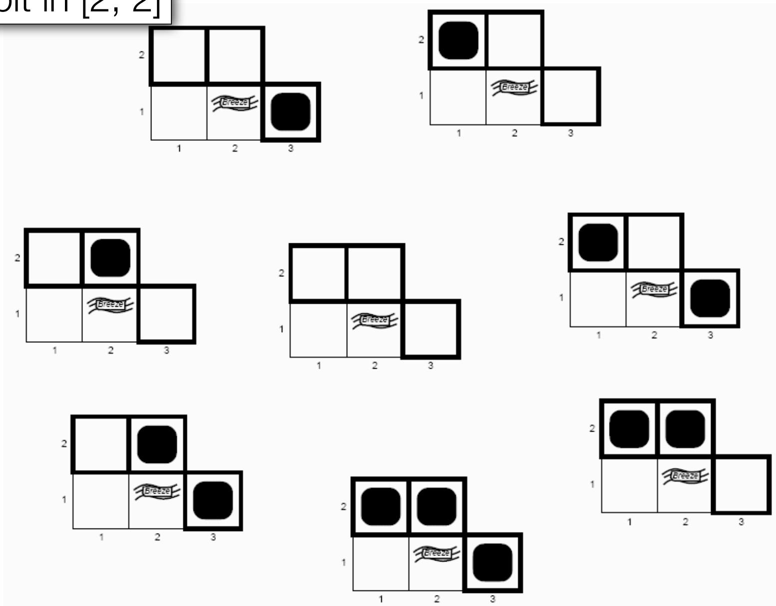
- *Reminder α ⊨ ß; ß follows logically from α; M(α) ⊆ M(ß)*
- α = (x = 0), ß = (xy = 0)
- α = (AI lectures on only Wednesday), ß = (No AI lectures on the weekends)
- α = (dogs have tails), ß = (Fido has a tail)
- α = (girls like flowers; Rachel is a girl), ß = (Rachel likes flowers)
- Everyone in class give their own example



α = KB
ß = No pit in [1, 2]



α = KB
β = No pit in [2, 2]





## Entailment

- This shows how entailment can be used to derive conclusions about the world
  - Performing *logical inference*
- Model checking
  - Generate all possible models
    - Must be a finite number of models
  - Check if hypothesis is true



## Inference

- KB $\vdash_i$ α
  - α is derived from KB by inference algorithm *i*
  - A *sound* inference algorithm only derives entailed sentences
  - A *complete* inference algorithm can derive any entailed sentence
- Model checking is sound & complete (when applicable)



## Propositional Logic

- Simple form of logic
- Can seem limited, but more complex forms of logic can be reduced to propositional logic

## Propositional Logic: Symbols

- Not: ¬
- And: ∧
- Or: ∨
- Implies: ⇒ or →
- If and only if: ⇔



## Prop. Logic Syntax

- *Sentence* → *AtomicSentence* | *ComplexSentence*
- *AtomicSentence* → *True* | *False* | *P* | *Q* | *R* | …
- *Complex Sentence* → (*Sentence*) | [*Sentence*]

  | ¬ *Sentence* | *Sentence* ∧ *Sentence*

  | *Sentence* ∨ *Sentence* | *Sentence* ⇒ *Sentence*

  | *Sentence* ⇔ *Sentence*
- Operator precedence: ¬, ∧, ∨, ⇒, ⇔



## Prop. Logic Semantics

- A model fixes the values of all variables to *true* or *false*
- True/False are always True/False
- Variables have their values defined in a model
- ¬P is true iff P is false in model
- P∧Q is true iff P and Q are both true in model
- P∨Q is true iff P or Q or both true in model
- P⇒Q is true iff P is false or P&Q are both true in model
- P⇔Q is true iff P&Q have the same values in model



## Semantics

- ⇒ and ⇔ not strictly needed
  - A⇒B is the same as ¬ A ∨ B
  - A⇔B is the same as (A⇒B)∧(B⇒A)

## Task

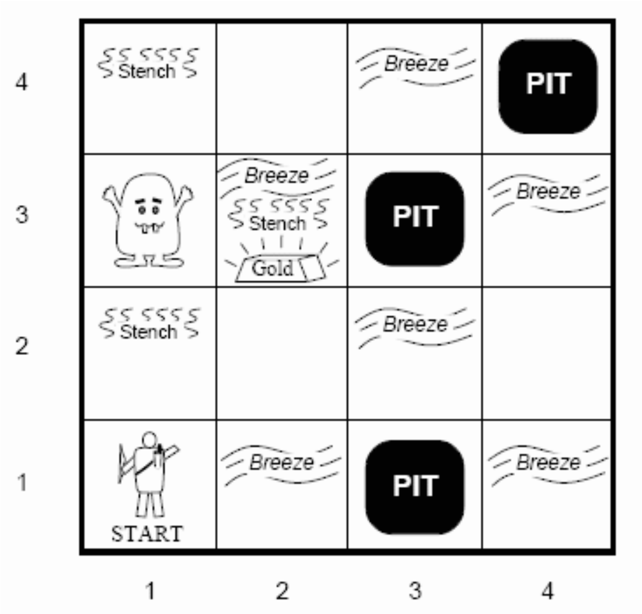
- Can our agent safely walk to (1, 2).
- Solution Steps:
  - Build KB (α)
  - Build Query (ß)
  - Test if α ⊨ ß
    - Using model checking



## Construct WW KB

- $P_{x, y}$ is true if there is a pit in [x, y]
- $W_{x, y}$ is true if there is a wumpus in [x, y]
- $B_{x, y}$ is true if the agent perceives breeze in [x, y]
- $S_{x, y}$ is true if the agent perceives stench in [x, y]








## WW KB

# WW KB

- There is no pit in [1, 1]



# WW KB

- There is no pit in [1, 1]
  - $R_1$: $\neg P_{1,1}$



# WW KB

- There is no pit in [1, 1]
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- A square is breezy iff there is a pit in a neighboring square



# WW KB

- There is no pit in [1, 1]
  - $R_1$: $\neg P_{1,1}$
- A square is breezy iff there is a pit in a neighboring square
  - $R_2$: $B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

## WW KB

- There is no pit in [1, 1]
  - $R_1$: $\neg P_{1,1}$
- A square is breezy iff there is a pit in a neighboring square
  - $R_2$: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
  - $R_3$: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$



## WW KB

- There is no pit in [1, 1]
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- Percepts:



## WW KB

- There is no pit in [1, 1]
  - $R_1$: $\neg P_{1,1}$
- A square is breezy iff there is a pit in a neighboring square
  - $R_2$: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
  - $R_3$: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
- Percepts:
  - $R_4$: $\neg B_{1,1}$



## WW KB

- There is no pit in [1, 1]
  - $R_1$: $\neg P_{1,1}$
- A square is breezy iff there is a pit in a neighboring square
  - $R_2$: $B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$
  - $R_3$: $B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
- Percepts:
  - $R_4$: $\neg B_{1,1}$
  - $R_5$: $B_{2,1}$

## Prop. Logic: Simple inference

- How many variables? How many models?
- In how many is KB true?



## Selection of possible models

| $B_{11}$ | $B_{21}$ | $P_{11}$ | $P_{12}$ | $P_{21}$ | $P_{22}$ | $P_{31}$ | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | KB |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F | F | F | F | F | F | F | | | | | | |
| F | T | F | F | F | F | T | | | | | | |
| F | T | F | F | F | T | F | | | | | | |
| F | T | F | F | F | T | T | | | | | | |



## Simple model checking

- How could we turn this into an algorithm?
- What is the running time?



Homework: 7.14(a)