Introduction to Artificial Intelligence
COMP 3501 / COMP 4704-4
Lecture 8: First Order Logic

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Lecture Overview

- First order logic (FOL)
- Inference in FOL

First Order Logic

- FOL is closer to natural languages that prop. logic
- FOL contains:
  - Objects (Constants):
    - people, locations, etc
  - Relations (between objects):
    - next to(C_{11}, C_{12}), older(a, b), father(a, b)
  - Functions (relation which “returns” object):
    - father_of(b), plus(one, two), leader(USA)

FOL

- Propositional logic reduces everything to true or false
- In FOL relations between objects are true or false (do or do not hold)
  - Objects
  - Relations
  - Functions -> map to objects

Prop. Logic Syntax

- *Sentence* → *AtomicSentence* | *ComplexSentence*
- *AtomicSentence* → *True* | *False* | *P* | *Q* | *R* | …
- *Complex Sentence* → (*Sentence*) | [*Sentence*]

  | ¬ *Sentence* | *Sentence* ∧ *Sentence*

  | *Sentence* ∨ *Sentence* | *Sentence* ⇒ *Sentence*

  | *Sentence* ⇔ *Sentence*

- Operator precedence: ¬, ∧, ∨, ⇒, ⇔

FOL syntax

- *Sentence* → *AtomicSentence* | *ComplexSentence*
- *AtomicSentence* → *Predicate* | *Predicate(Term…)* | *Term* = *Term*
- *Complex Sentence* → (*Sentence*) | [*Sentence*]

  | ¬ *Sentence* | *Sentence* ∧ *Sentence*

  | *Sentence* ∨ *Sentence* | *Sentence* ⇒ *Sentence*

  | *Sentence* ⇔ *Sentence* | $Quantifier_{Variable}$, *Sentence*

- Operator precedence: ¬, ∧, ∨, ⇒, ⇔

FOL syntax

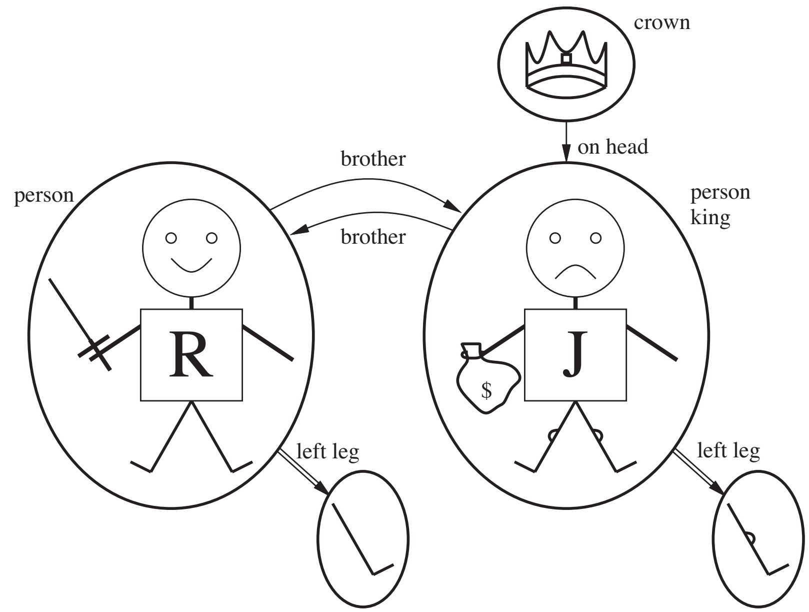
- *Term* → *Function(Term, …)* | *Constant* | *Variable*
- *Quantifier* → ∀ | ∃
- *Constant* → *A* | P_1 | *Fred* | …
- *Variable* → *a* | *x* | *s* | …
- *Predicate* → *True* | *False* | *After* | *Loves* | *Raining*
- *Function* → *Mother* | *LeftLeg* | …

FOL semantics

- Semantics are somewhat flexible
  - In C++ you can overload the + operator to be *
  - Similarly, we can define objects in a way that is nonsensical in the real world
  - *Intended Interpretation* is when objects represent the names they have in the real world

FOL Models

- A model in propositional had the truth value of each variable
- A model in FOL has:
  - All objects
  - The relations between objects
  - Functions on objects



FOL Models

- A model contains one set of interpretations
- What about all models?
  - Not only all interpretations on a fixed set of objects
  - Also all numbers of objects and all interpretations of relations between them

Sentences

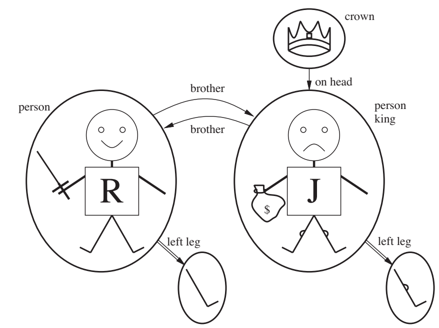
- *Atomic sentences* state facts
  - *Married(Father(Richard), Mother(John))*
  - The sentence is true in a model if the relations hold in the model
- Complex sentences work like prop. logic
  - *King(Richard)* ∨ *King(John)*

Universal Quantifier (∀)

- Quantifiers let us solve the problem of making general statements about the world
  - All kings are persons
  - $\forall_x \text{ King}(x) \Rightarrow \text{Person}(x)$
  - For all *x*, if *x* is a king, then *x* is a person
- Lower case letters are variables

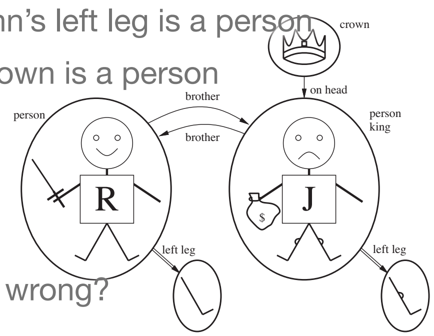
Universal Quantifier (∀)

- Formally, when quantifying a variable we need to look at all the extended interpretations of that variable
  - eg $\forall_x$, what are all things that *x* can be?
- $\forall_x \text{ King}(x) \Rightarrow \text{Person}(x)$
  - This is true if it is true for all interpretations of *x*



Universal Quantifier (∀)

- Richard is a king ⇒ Richard is a person
- John is a king ⇒ John is a person
- Richard's left leg is a king ⇒ Richard's left leg is a person
- John's left leg is a king ⇒ John's left leg is a person
- The crown is a king ⇒ The crown is a person
- Note that this works because of our definition of ⇒
- Why is $\forall_x \text{ King}(x) \wedge \text{Person}(x)$ wrong?



Existential quantifier (∃)

- Universal quantifiers make statements about all objects
- Existential quantifiers claim that at least one object has a given property
  - ∧ is the natural connector to use with ∃

Existential quantifier (∃)

- $\exists_x$ Crown(x) ∧ OnHead(x, John)
  - Richard is a crown ∧ Richard is on John's head
  - John is a crown ∧ John is on John's head
  - Richard's left leg is a crown ∧ Richard's left leg is on John's head
  - John's left leg is a crown ∧ John's left leg is on John's head
  - The crown is a crown ∧ The crown is on John's head
- Why is $\exists_x$ Crown(x) ⇒ OnHead(x, John) wrong?

Nested quantifiers

- Can nest multiple quantifiers together
  - $\forall_{x,\, y}$ brother(x, y) ⇔ brother(y, x)
  - With the same quantifier, order doesn't matter
- What is the difference?
  - $\forall_x \exists_y$ loves(x, y)
  - $\exists_x \forall_y$ loves(x, y)
- Avoid re-using variables in quantifiers

Negating quantifiers

- How do we write everyone likes ice cream?

Negating quantifiers

- De Morgan's rules for quantification:
  - $\forall_x P \equiv \neg \exists_x \neg P$
  - $\exists_x P \equiv \neg \forall_x \neg P$
  - $\forall_x \neg P \equiv \neg \exists_x P$
  - $\neg \forall_x P \equiv \exists_x \neg P$

Equality

- Equality indicates that two terms refer to the same objects
  - Father(John) = Henry
- Often used with multiple existential variables:
  - Richard has at least two siblings
  - $\exists_{x,\, y}$ Sibling(x, Richard) $\wedge$ Sibling(y, Richard)
  - $\exists_{x,\, y}$ Sibling(x, Richard) $\wedge$ Sibling(y, Richard) $\wedge \neg$ (x = y)

First-Order Logic Examples

- All cows eat grass.
- Some cows don't eat grass
- Every good boy deserves fudge.
- My dog likes popcorn.

Lecture overview

- Continue practicing FOL
- Inference in FOL
- Midterm review

Review: universal quantification

- A statement with universal quantification (∀) is considered true iff:
  - For all variable substitutions the statement is true
  - $[\ \forall_x\ (\text{awake}(x) \Rightarrow \text{alive}(x))^1\ ]^2$
  - 2 is true iff 1 is true for all substitutions in 1

Review: universal quantification

- Statements with universal quantification *often,* but not always, involve implications or $\vee$
  - Everything is an animal, mineral or vegetable.
    - $\forall_x$ animal(x) $\vee$ mineral(x) $\vee$ vegetable(x)
  - All dogs like bones
    - $\forall_x$ dog(x) $\Rightarrow$ likes(x, Bones)
  - Everything is valuable
    - $\forall_x$ valuable(x)

Review: existential quantification

- A statement with existential quantification ($\exists$) is considered true iff:
  - For at least one variable substitution the statement is true
  - $[\ \exists_x\ (\text{awake}(x) \wedge \text{alive}(x))^1\ ]^2$
  - 2 is true iff 1 is true for some substitutions in 1

Review: existential quantification

- Existential quantification almost never is used with implications
  - $\exists_x$ boy(x) $\Rightarrow$ sleeps(x)
  - $\exists_x$ $\neg$boy(x) $\vee$ sleeps(x)
    - As long as something it the world is not a boy, this holds.
    - “There is either something that is not a boy, or there is a boy that sleeps.”

First-Order Logic Examples

- All cows eat grass.
- Some cows don’t eat grass
- Every good boy deserves fudge.
- My dog likes popcorn.

More FOL examples

- The only two certainties in life are death and taxes.
- The coldest winter I ever spent was a summer in San Francisco.
- You can fool some of the people all of the time, and all of the people some of the time, but you cannot fool all of the people all of the time.

Homework: 9.10