Lecture 8: Multi-Player Games

AI For Traditional Games
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Review: Multi-player game tree

- All nodes are max nodes
- Scores/payoffs are *n*-tuples, with a value for each player
  - Assume that players strictly want to increase their own payoff and don't care about minimizing/ maximizing other players payoff

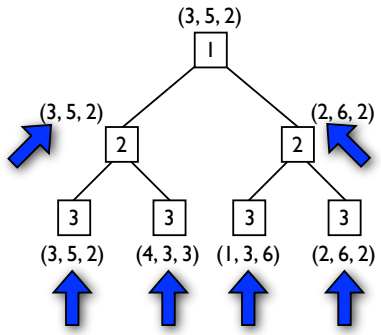
Zero-sum / constant sum

- Have been looking at two-player zero-sum games
  - Multi-player equivalent is constant-sum
  - eg (3, 4, 2) (sum = 9)
    - Subtract 3 from each player: (0, 1, -1) (zero sum)

Max^n

```
Max^n(GameState *g, Player *p, depth)
    if (game over or depth == 0) return p->eval(g)
    best ← nil
    for each successor s in 1… # successors
        ApplyMove(s)
        tmp = Max^n(g, p, depth-1)
        UndoMove(s)
        if (tmp[g->currPlayer()] > best[g->currPlayer())
            best = tmp;
    return best
```

Sample Tree

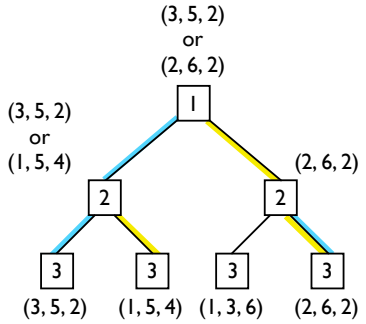


Max^n in equilibrium?

- Max^n generalizes minimax for *n* players
- All players are making locally maximal choices
  - If any player changes their strategy
    - They must get a lower score
    - Thus, and equilibrium
  - Note that the strategy change is local in the tree
    - Other players cannot adjust their strategies accordingly higher in the tree; those moves already made

Max^n tie-breaking & implications

- Each different tie-breaking strategy leads to a different strategy
- Blue / yellow strategies are separate equilibria
  - Cannot be mixed to produce an equilibrium strategy

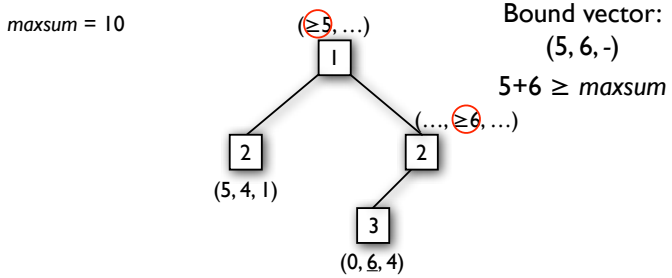


Can we prune in max^n?

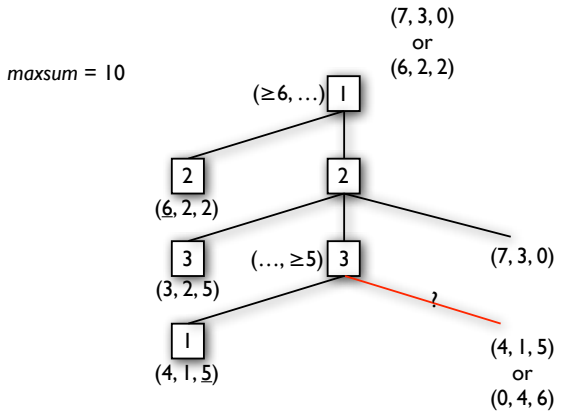
- With no restrictions on players score, no
- What restrictions do we have?
  - maxsum, minsum, maxp, minp
  - Assume maxsum = minsum (constant sum)
  - Assumpe minp = 0

Shallow pruning

- First analyzed by (Korf, 91)



Deep pruning



Deep Pruning

- Deep pruning can fail in general
  - Bounds at the bottom of the tree cannot be the max^n value of the tree
  - But, propagation of effects can travel arbitrarily far up the tree
  - Zipper effect -- order of teeth at the bottom determines which one comes out on top

Last Branch Pruning (first attempt)

- Deep pruning fails because there are additional moves which can influence the values in the tree
- What if there were no other moves left?
  - No moves to influence propagation

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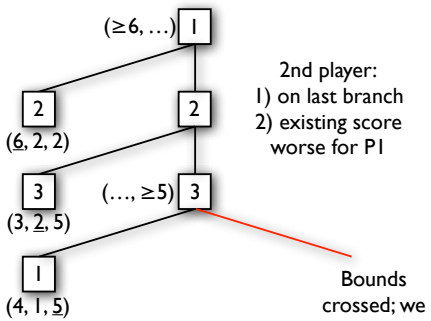
Last Branch Pruning (First attempt)

maxsum = 10



2nd player:
1) on last branch
2) existing score worse for P1

Bounds crossed; we can prune